CBSE Class 10 Mathematics

Important Question

Chapter 3

Pair of Linear Equations In Two Variables

Like the crest of a peacock so is mathematics at the head of all knowledge.

1. **At a certain time in a deer park, the number of heads and the number of legs of deer and human visitors were counted and it was found there were 39 heads & 132 legs. Find the number of deer and human visitors in the park.**

   Ans: Let the no. of deers be x

   And no. of humans be y

   ASQ:

   x + y = 39 …….. (1)

   4 x + 2 y = 132 ……. (2)

   Multiply (1) and (2)

   On solving, we get …

   x = 27 and y= 12

   ∴ No. of deers = 27 and No. of humans = 12

2. **Solve for x, y**

   a. $\frac{x+y-8}{2}=\frac{x+2y-14}{3}=\frac{3x+y-12}{11}$

   Ans: $\frac{x+y-8}{2}=\frac{x+2y-14}{3}=\frac{3x+y-12}{11}$

   $\frac{x+y-8}{2}=\frac{x+2y-14}{3}$

   On solving, we will get….y= 6

   $\frac{x+y-8}{2}=\frac{x-2}{2}=\frac{x+2y-14}{3}$

   On solving , we will get….

   x = 2

   b. **7(y + 3) 2(x + 2) = 14, 4 (y 2) + 3(x 3) = 2**

   Ans: 7(y + 3) – 2 (x+ 2) = 14 ………. (1)

   4(y– 2) + 3(x – 3) = 2 ………. (2)

   From (1) 7y +21 – 2x – 4 = 14

   On solving, we will get….

   2x – 7y – 3 = 0 ………. (3)

From (2) 4y – 8 + 3x -9 =2

On solving, we will get....

3x + 4y – 19 =0 .... (4)

2x– 7y – 3

3x + 4y – 19

Substitute this, to get y = 1 and x = 5

∴ x = 5 and y = 1

c. **(a+2b)x + (2a b)y = 2, (a 2b)x + (2a +b)y = 3**

Ans: 2ax + 4ay = y,

we get 4bx – 2by = -1

2ax+ 4ay = 5 4bx– 2by = - 1

Solve this, to get $y = \frac{10b+a}{10ab}$

Similarly, we can solve for x

d. $\frac{x}{a} + \frac{y}{b} = a + b, \frac{x}{2} + \frac{y}{b^2} = 2$; **a ≠ 0, b ≠ 0**

Ans: $\frac{x}{a} + \frac{y}{b} = a + b$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

$$\frac{xb+ya}{ab} = a + b$$

$$\frac{xb^2+ya^2}{a^2b^2} = 2$$

On solving, we get ... x= a^2 and y= b^2

e. **$2^x + 3^y = 17, 2^{x+2}$ 3 $^{y+1} = 5$**

Ans: $2^x + 3^y = 17, 2^{x+2} - 3^{y+1} = 5$

Let 2^x be a and 3^y be b

$2^x + 3^y = 17$

a + b = 17 .... (1)

$2^{x+2} - 3^{y+1} = 5$

4a – 3b = 5 .... (2)

on solving, we get..... a = 8

from (1)

a + b = - 17

∴ b = 9, a = 8

⇒ x = 3, y = 221

f. **If** $\frac{4x-3y}{7x-6y} = \frac{4}{13}$, **Find** $\frac{x}{y}$

Ans: $\frac{4x-3y}{7x-6y} = \frac{4}{13}$

On dividing by y, we get $\frac{x}{y} = \frac{5}{8}$

g. **41x + 53y = 135, 53x +41y =147**

Ans: 41x + 53 y = 135, 53 x + 41 y = 147

Add the two equations:

Solve it, to get ... x + y = 3 .... (1)

Subtract:

Solve it, to get, ... x – y = 1 .... (2)

From (1) and (2)

x + y = 3

x – y = 1

on solving, we get ... x = 2 and y = 1

3. **Find the value of p and q for which the system of equations represent coincident lines**

2x +3y = 7, (p+q+1)x +(p+2q+2)y = 4(p+q)+1

Ans: $a_1 = 2$, $b_1 = 3$, $c_1 = 7$

$a_2 = p + q + 1$, $b_2 = p + 2q + 2$, $c_2 = (p + q) + 1$

For the following system of equation the condition must be

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{p+q+1} = \frac{3}{p+q+2} = \frac{7}{4(p+q)+1}$

$\Rightarrow \frac{2}{p+q+1} = \frac{7}{4(p+q)+1}$

7p +14q + 14 = 12p + 12q + 3

= 5p - 2q - 11 = 0 .... (2)

p + q + - 5 = 0

5p - 2q - 11 = 0

From (1) and (2)

5p + 5q - 25 = 0

5p - 2q - 11 = 0

Solve it, to get q = 2

Substitute value of q in equation (1)

p + q - 5 = 0

On solving we get, p = 3 and q = 2

4. **Students are made to stand in rows. If one student is extra in a row there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of students in the class.**

Ans: No. of rows be y

Let the number of students be x

Number of students in the class will be = xy

One student extra, 2 rows less

(x + 1) (y – 2) = xy

xy – 2x + y -2 = xy

-(-2x + y -2) = 0

+2x – y = -2 .... (1)

One student less, three more rows

(x - 1) (y + 3) = xy

xy + 3x - y -3 = xy

3x - y = 3 ....(2)

From (1) & (2)

2x – y = -2 × 3

3x - y = 3 × -2

Solve it, to get ... y = 12 and x = 5

∴ Number of student = xy

= 12 × 5

= 60 students

5. **The larger of two supplementary angles exceeds the smaller by 18^0, find them.**

Ans:

$$x + y = 180^\circ$$

$$x - y = 18^\circ$$

2x = 198

x = 198 / 2 = x = 99^0

x + y = 1800

y = 180 – 99

$y = 81^0$

6. **A train covered a certain distance at a uniform speed. If the train would have been 6km/hr faster, it would have taken 4 hours less than the scheduled time. And if the train were slower by 6km/hr, it would have taken 6 hours more than the scheduled time. Find the distance of the journey.**

Ans: Let the speed of the train by x km/hr

And the time taken by it by y

Now distance traveled by it is x x y = xy

APQ:

I--- (x + 6) (y - 4) = xy

4x – 6y = -24

=> 2x – 3y = -12 ....(1)

II--- (x - 6) (y+ 6) = xy

6x – 6y= 36

=> x– y= 6 .... (2)

Solving for x and y we get y = 24, x = 30

So the distance =30 × 24 = 720 km

7. **A chemist has one solution which is 50% acid and a second which is 25% acid. How much of each should be mixed to make 10 litres of 40% acid solution.**

Ans: Let 50 % acids in the solution be x

Let 25 % of other solution be y

Total Volume in the mixture = x + y

A.P.Q:

x + y = 10 .... (1)

A.P.Q: $\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100} \times 10$

2x + y = 16 .... (2)

So x = 6 & y = 4

8. **The length of the sides of a triangle are $2x + \frac{y}{2}, \frac{5x}{3} + y + \frac{1}{2}$ and $\frac{2}{3}x + 2y + \frac{5}{2}$. If the triangle is equilateral. Find its perimeter.**

Ans: $2x + \frac{y}{2}$

$= \frac{4x+y}{2}$ .... (1)

$= \frac{10x+6y+3}{6}$ .... (2)
$\frac{2}{3}x + 2y + \frac{5}{2}$
$= \frac{4x+12y+15}{6}$ .... (3)

APQ:

$\frac{4x+y}{2} = \frac{10x+6y+3}{6} = \frac{4x+12y+15}{6}$

24x + 6y= 20x + 12y + 6

2x – 3y = 3 ---------(4)

$\frac{4x+y}{2} = \frac{4x+12y+15}{6}$

24x + 6y = 8x + 24y + 30

Solve it,

To get 8x – 9y = 15 .... (5)

Solve it,

To get x = 3

Substitute value of x in (4)

2x– 3y = 3

Solve it,

To get y = 1

So the values of x = 3 and y = 1

$2x + \frac{y}{2} = 6.5cm$

Perimeter = 6.5 cm + 6.5 cm + 6.5 cm

Perimeter = 19. 5cm

∴ the perimeter of the triangle is 19.5 cm

9. **In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes & this later number was equal to twice his majority over B. If there were 18000 persons on the electoral roll. How many voted for B.**

Ans: Let x and y be the no. of votes for A & B respectively.

The no. of persons who did not vote = (18000 – x – y)

APQ:

x = 2(18000 – x – y)

⇒ 3x + 2y = 36000 .... (1)

&

(18000 – x – y) = (2) (x – y)

⇒ 3x – y = 18000 .... (2)

On solving we get, y = 6000 and x = 8000

Vote for B = 6000

10. **When 6 boys were admitted & 6 girls left the percentage of boys increased from 60% to 75%. Find the original no. of boys and girls in the class.**

Ans: Let the no. of Boys be x

Girls be y

Total = x + y

APQ:

$\frac{x}{x+y} = \frac{60}{100}$ .... (1)

$\frac{x+6}{(x+6)(y-6)} = \frac{75}{100}$

On solving we get,

x = 24 and y = 16.

11. **When the son will be as old as the father today their ages will add up to 126 years. When the father was old as the son is today, their ages add upto 38 years. Find their present ages.**

Ans: let the son's present age be x

Father's age be y

Difference in age (y – x)

Of this difference is added to the present age of son, then son will be as old as the father now and at that time, the father's age will be [y + (y – x)]

APQ:

[x + (y – x)] + [y (y – x)] = 126

[y + (x – y)] + [x + (x – y)] = 38

Solving we get the value of x and y

12. **A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30km at half speed in 5 hours. If the breakdown had occurred 10km farther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.**

Ans: Let x be the place where breakdown occurred

y be the original speed

$\frac{x}{y} + \frac{30-x}{\frac{y}{2}} = 5$

$\frac{x+10}{y}+\frac{30-(x+10)}{\frac{y}{2}}=4$

$\frac{x}{y}+\frac{60-2x}{y}=5$

On solving, we get, x = 10 km and y = 10 km/h

13. **The population of the village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of the year. Find the number of males and females in the village.**

Let the number of Males be x and females be y

Ans: x + y = 5000

$x+\frac{5}{100}x+y+\frac{3y}{100}=5202$ ...1

⇒5x+3y = 20200 ...2

On solving 1 & 2 we get x= 2600 y=2400

No. of males = 2600

No. of females = 2400

CBSE Class 10 Mathematics

Important Questions

Chapter 3

Pair of Linear Equations in Two Variables

1. **Graphically, solve the following pair of equations $2x + y = 6$ and $2x - y + 2$ Find the ratio of the areas of the two triangles formed by the lines representing these equations with the X-axis and the lines with the Y-axis.**
   Ans- 4:1
2. **Determine graphically, the vertices of the triangle formed by the lines $y = 3$, $x = 3y$, and $x + y = 8$**
   Ans- (0,0), (4,4), (6,2)
3. **Draw the graphs of the equations $x = 3$, $x = 5$, and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the X-axis.**
   Ans- 8 sq units.
4. **Gandharv had some bananas and he divide them into two lots A and B. He sold the first lot at the rate of Rs. 2 for 3 bananas and the second lot at the rate of Rs.1 per banana and got a total of Rs. 400. If he had sold the first lot at the rate of Rs. 1 per banana and the second lot at the rate of Rs. 4 for 5 bananas. His total collection would had been Rs. 460 Find the total number of banana he had.**
   Ans- 500
5. **Sarthak invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum respectively. He received Rs. 1860 annual interest. However, had he interchange the amount of investment in two schemes, he would have received Rs. 20 as more as annual interest. How much money did he invest in each school?**
   Ans- 12000 and 10000.
6. **A railway half ticket cost half the full fare but the reservation charges are the same on half ticket as on a full ticket. One reserved first class ticket from station A to B cost Rs 2530. Also one reserved first class ticket and one reserved first class half ticket from station A to b Costs Rs. 3810. Find the full first class fare from station A to B and also the reservation charges for a ticket.**

Ans- 2500 and 30

7. **A shopkeeper sells a saree at 8% profit and the sweater at 10% discount, thereby, getting a sum of Rs. 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got 1028 Rs. Than find the cost of saree and list price (price before discount) of the sweater**

   Ans- 600 and 400

8. **A two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.**

   Ans- 83

9. **A motor boat can travel 30 km upstream and 28 km downstream in 7h. It can travel 21km upstream and return in 5h. Find the speed of the boat in still water and the speed of the Stream.**

   Ans- 10km/h and 4km/h

CBSE Class 10 Mathematics

Important Questions

Chapter 3

Pair of Linear Equations

1 Marks Questions

1. A pair of linear equation in two variables which has a common point i.e., which has only one solution is called a

(a) Consistent pair

(b) Inconsistent pair

(c) Dependent pair

(d) None of these

Ans. a) Consistent pair

2. If a pair of linear equation $a_1x+b_1y+c_1=0$ **and** $a_2x+b_2y+c_2=0$ **represents coincident lines, then**

(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(d) None of these

Ans. (c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

3. The value of 'k' for which the system of equation 2x + 3y = 5 and 4x + ky = 10 has infinite number of solutions is

(a) k = 1

(b) k = 3

(c) k = 6

(d) k = 0

Ans. (c) k = 6

4. If the system of equation 2x + 3y = 7 and 29x + (a + b) y = 28 has infinitely many solutions, then

(a) a = 2b

(b) b = 2a

(c) a + 2b = 0

(d) 2a + b = 0

Ans. (b) b = 2a

5. If am ≠ bl, then the system of equation ax + by = c and lx + my = n

(a) has a unique solution

(b) has no solution

(c) has infinitely many solution

(d) may or may not have a solution

Ans. (a) has a unique solution

6. The graphical representation of the linear equation y – 5 = 0 is

(a) a line

(b) a point

(c) a curve

(d) None of these

Ans. (a) a line

7. A system of simultaneous linear equations is said to be inconsistent, if it has

(a) One solution

(b) Two solutions

(c) Three solutions

(d) No solution

Ans. (d) No solution

8. The system of equation $2x + 3y - 7 = 0$ and $6x + 5y - 11 = 0$ has

(a) unique solution

(b) No solution

(c) Infinitely many solutions

(d) None of these

Ans. (a) unique solution

9. The value of 'k' for which the system of equation $x + 2y - 3 = 0$ and $5x + ky + 7 = 0$ has no solution is

(a) $k = 10$

(b) $k = 6$

(c) $k = 3$

(d) k = **1**

Ans. (a) k = 10

10. The equation ax^{n} + by^{n} + c = 0 represents a straight line if

(a) n $\geq$ **1**

(b) n $\leq$ **1**

(c) n=1

(d) None of these

Ans. (c) n=1

11. The value of 'k' for which the system of equation kx – y = 2 and 6x – 2y = 3 has a unique solution is

(a) k = **3**

(b) k $\neq$

(c) k = **0**

(d) k $\neq$ **0**

Ans. (b) k $\neq$ 3

12. The value of 'k' for which the system of equations x + 2y = 5 and 3x + ky + 15 = 0 has no solution, if

(a) k = **6**

(b) k = **–6**

(c) –k = $\frac{3}{2}$

(d) **None of these**

Ans. (a) $k = 6$

13. In the equation $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then the equation will represents

(a) coincident lines

(b) parallel lines

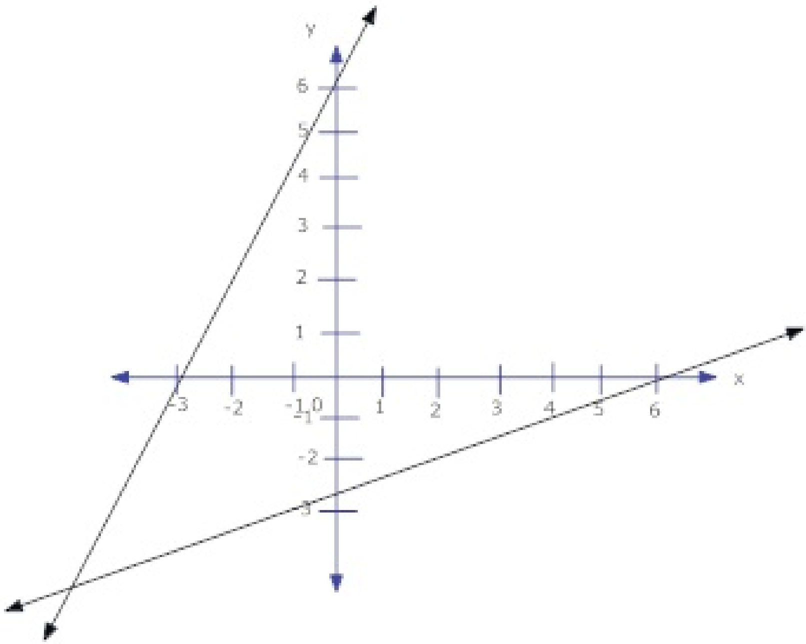
(c) intersecting lines

(d) None of these

Ans. (c) intersecting lines

14. Solve graphically $2x - 3y + 13 = 0$ and $3x - 2y + 12 = 0$

Ans. $2x - 3y + 13 = 0$



$3x - 2y + 12 = 0$

$$when\ x = \frac{13 + 3y}{2}$$

$$when\ y = \frac{3x + 12}{2}$$

x	0	-3
y	6	3

X	13/2	5
y	0	-1

CBSE Class 10 Mathematics

Important Questions

Chapter 3

Pair of Linear Equations

2 Marks Questions

1. Half the perimeter of a rectangle garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Ans. Let length of rectangular garden = x metres

Let width of rectangular garden = y metres

According to given conditions, perimeter = 36 m

$\Rightarrow$ x + y = 36 ......(i)

And x = y+4

$\Rightarrow x - y = 4$ ....(ii)

Adding eq. (i) and (ii),

2x = 40

$\Rightarrow$ x = 20 m

Subtracting eq. (ii) from eq. (i),

2y = 32

$\Rightarrow$ y = 16 m

Hence, length = 20 m and width = 16 m

2. Draw the graphs of the equations x – y+1=0 and 3x+2y – 12=0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and

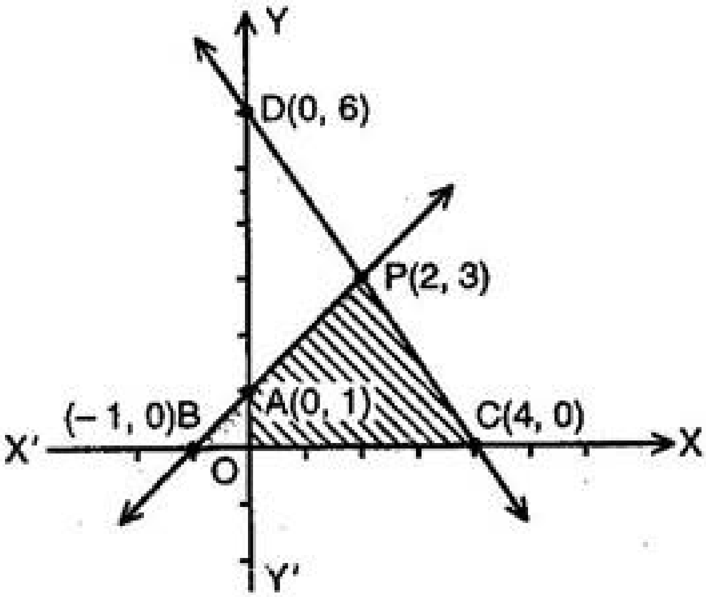
shade the triangular region.

Ans. For equation x – y + 1 = 0, we have following points which lie on the line

x	0	-1
y	1	0

For equation 3x + 2y – 12 = 0, we have following points which lie on the line

x	4	0
y	0	6



We can see from the graphs that points of intersection of the lines with the x–axis are (–1, 0), (2, 3) and (4, 0).

3. The age of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Ans. Let the age of Ani and Biju be x years and y years respectively.

Age of Dharam = 2x years and Age of Cathy = $\frac{y}{2}$ years

According to question,

$x - y = 3$ ... (1)

And $2x - \frac{y}{2} = 30$

$\Rightarrow 4x - y = 60$ ... (2)

Subtracting (1) from (2), we obtain:

$3x = 60 - 3 = 57$

$\Rightarrow$ x = Age of Ani = 19 years

Age of Biju = 19 – 3= 16 years

Again, According to question, $y - x = 3$ ... (3)

And $2x - \frac{y}{2} = 30$

$\Rightarrow 4x - y = 60$ ... (4)

Adding (3) and (4), we obtain:

$3x = 63$

$\Rightarrow x = 21$

Age of Ani = 21 years

Age of Biju = 21 + 3 = 24 years

4. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II]

Ans. Let the money with the first person and second person be Rs x and Rs y respectively.

According to the question,

$x + 100 = 2(y - 100)$

$\Rightarrow x + 100 = 2y - 200$

$\Rightarrow x - 2y = 300$ ... (1)

Again, $6(x - 10) = (y + 10)$

$\Rightarrow 6x - 60 = y + 10$

$\Rightarrow 6x - y = 70$ ... (2)

Multiplying equation (2) by 2, we obtain:

$12x - 2y = 140$ ... (3)

Subtracting equation (1) from equation (3), we obtain:

$11x = 140 + 300$

$\Rightarrow 11x = 440$

$\Rightarrow x = 40$

Putting the value of x in equation (1), we obtain:

$40 – 2y = –300$

$\Rightarrow 40 + 300 = 2y$

$\Rightarrow 2y = 340$

$\Rightarrow y = 170$

Thus, the two friends had Rs 40 and Rs 170 with them.

5. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans. Let the number of rows be x and number of students in a row be y.

Total number of students in the class = Number of rows x Number of students in a row = xy

According to the question,

Total number of students = $(x – 1)(y + 3)$

$\Rightarrow xy = (x – 1)(y + 3)$

$\Rightarrow xy = xy – y + 3x – 3$

$\Rightarrow 3x – y – 3 = 0$

$\Rightarrow$ 3x – y = 3 ... (1)

Total number of students = (x + 2) (y – 3)

$\Rightarrow$ xy = xy + 2y – 3x – 6

$\Rightarrow$ 3x – 2y = –6 ... (2)

Subtracting equation (2) from (1), we obtain:

y = 9

Substituting the value of y in equation (1), we obtain:

3x – 9 = 3

$\Rightarrow$ 3x = 9 + 3 = 12

$\Rightarrow$ x = 4

Number of rows = x = 4

Number of students in a row = y = 9

Hence, Total number of students in a class = xy = 4 x 9 = 36

6. Find the values of α and β for which the following system of linear equations has infinite number of solutions, 2x + 3y = 7, 2α x + $(\alpha+\beta)$ y = 28.

Ans. $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}(\text{Infinite solution})$

$$\frac{2}{2\alpha}=\frac{3}{\alpha+\beta}=\frac{-7}{-28}$$

$\Rightarrow \alpha=4,$ and $\beta=8$

7. Find the condition for which the system of equations $\frac{x}{a}+\frac{y}{b}=c$ and bx + ay = 4ab (a, b $\neq$ 0) is inconsistent.

Ans. Inconsistent

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{1/a}{b}=\frac{1/b}{a}\neq\frac{c}{4ab}$$

i.e. $\frac{1}{ab}=\frac{1}{ab}\neq\frac{c}{4ab}$

or $c\neq 4$

8. Find the value of 'α' so that the following linear equations have no solution

$$(3\alpha+1)x+3y-2=0, (\alpha^2+1)x+(\alpha-2)y-5=0$$

Ans. No solution

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

i.e. $\frac{3\alpha+1}{\alpha^2+1}=\frac{3}{\alpha-2}\neq\frac{-2}{-5}$

$$3\alpha^2-6\alpha+\alpha-2=3\alpha^2+3$$

$$-5\alpha=5$$

or $\alpha=-1$

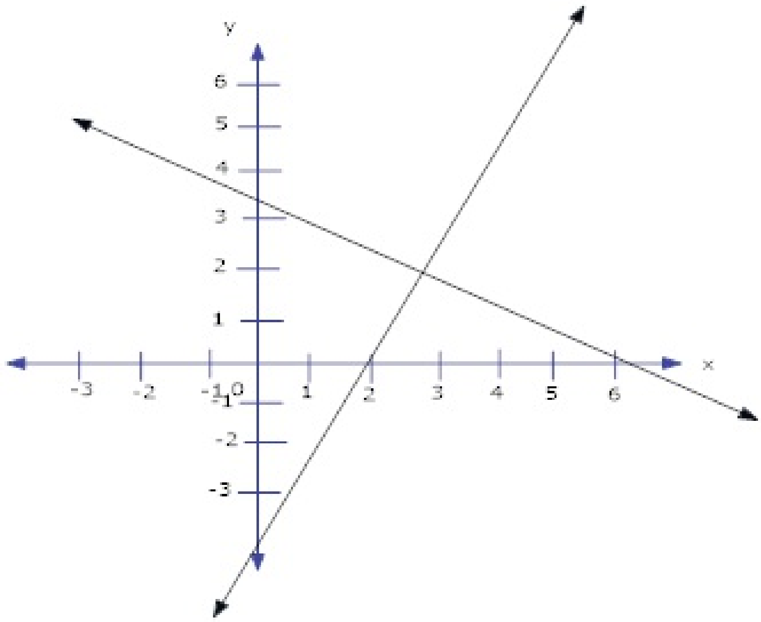
or $\frac{3}{\alpha-2}\neq\frac{2}{5}$

$\Rightarrow\ \alpha\neq\frac{19}{2}$

9. Solve for x and y: ax + by = a – b and bx – ay = a + b

Ans. $ax+by=a-b\ ]\times a$

$$bx-ay=a+b\ ]\times b$$



$$\begin{array}{r} a^2x + aby = a^2 - ab \\ bx - aby = ab + b^2 \\ \hline (a^2 + b^2)x = a^2 + b^2 \end{array}$$

$\Rightarrow x = 1$

$\therefore a + by = a - b$

$by = -b$

$y = -1$

$\therefore x = 1$

$y = -1$

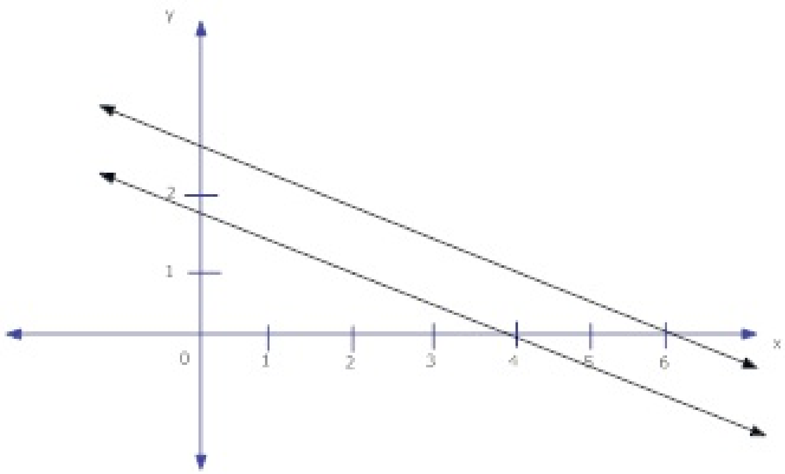
10. The path of a train A is given by the equation x + 2y – 4 = 0 and the path of another train B is given by the equation 2x + 4y – 12 = 0 represent this situation graphically.

Ans. $x + 2y - 4 = 0$

$2x + 4y - 12 = 0$

$when\ x = 4 - 2y$

x	4	2
y	0	1



$when\ x = \frac{12-4y}{2}$

x	6	2
y	0	2

11. For what value of 'α' the system of linear equations α .x + 3y = α - 3, 12x + α y = α has no solution.

Ans. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$i.e. \ \frac{\alpha}{12} = \frac{3}{\alpha} \neq \frac{\alpha - 3}{\alpha}$

$If \ \frac{\alpha}{12} = \frac{3}{\alpha} \Rightarrow \alpha^2 = 36$

or $\alpha = \pm 6 \rightarrow (i)$

$If \ \frac{3}{\alpha} \neq \frac{\alpha - 3}{\alpha}$

$\Rightarrow \alpha^2 - 3\alpha \neq 3\alpha$

or $\alpha^2 \neq 6\alpha$

or $\alpha = 0$ and $\alpha = 6 \rightarrow (ii)$

$\therefore$ from eq (i) and (ii)

The value of α is 6.

12. Find the values of 'a' and 'b' for which the following system of linear equations has infinite number of solutions. 2x + 3y = 7, (a + b + 1) x +(a + 2b + 2) y = 4 (a + b) +1

Ans. If infinite number of number of solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

or $\dfrac{2}{a+b+1} = \dfrac{3}{a+2b+2} = \dfrac{7}{4a+4b+1}$

If $\dfrac{2}{a+b+1} = \dfrac{3}{a+2b+2}$

$\Rightarrow a-b=1$

and if $\dfrac{3}{a+2b+2} = \dfrac{7}{4a+4b+1}$

$\Rightarrow 5a-2b=11$

on solving we get,

$a=3$ and $b=2$

13. Solve for 'x' and 'y' where x + y = a – b, ax – by = $a^2 + b^2$

Ans. $x+y=a-b$

and $ax-by=a^2+b^2$

$$\begin{aligned} bx+by &= ab-b^2 \\ ax-by &= a^2+b^2 \\ \hline (a+b)x &= a(a+b) \end{aligned}$$

$x=a$

$x+y=a-b$

$a+y=a-b$

$y=-b$

14. The cost of two kg of apples and 1 kg of grapes on a day was found to be Rs. 160. After a month the cost of 4 kg apples and 2 kg grapes is Rs. 300. Represent the situation algebraically and graphically.

Ans. Let the cost of one Kg of apple is x and one Kg of grapes is y.

According to question,

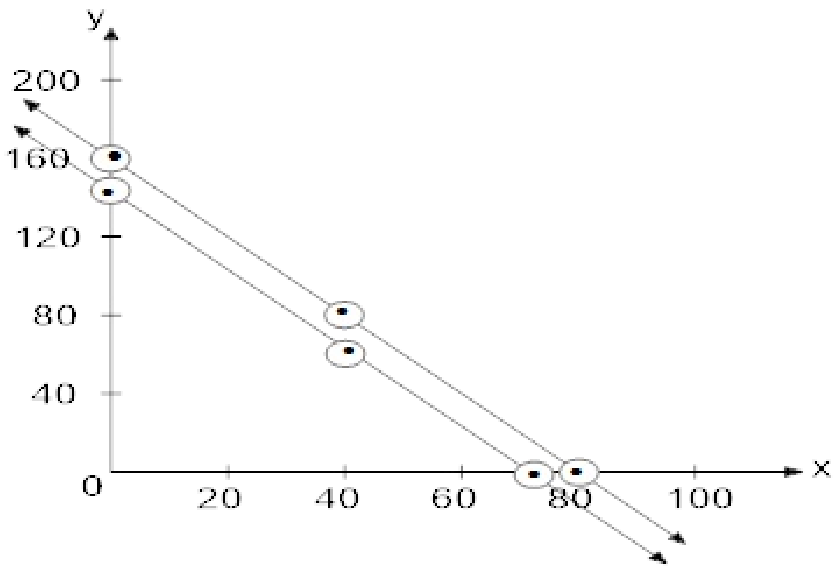
$2x + y = 160$ and $4x + 2y = 300$

$2x + y = 160$

x	0	80	40
Y	160	0	80

$4x + 2y = 300$

x	0	75	40
Y	150	0	70



15. Find the value of 'k' for which the system of equation kx + 3y = k – 3 and 12x + ky = k will have no solution.

Ans. $kx + 3y = k - 3$

$12x + ky = k$

The system has no solution.

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{k}{12} = \frac{3}{k} \neq \frac{k-3}{k}$

$k^2 = 36$

$\Rightarrow k = \pm 6$ (i)

If $\frac{3}{k} \neq \frac{k-3}{k}$

$3k \neq k^2 - 3k$

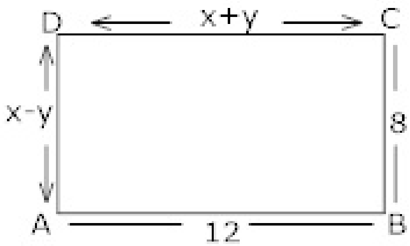
$k^2 - 6k \neq 0$

$k(k-6) \neq 0$

$k \neq 6 \ (ii)$

$\therefore \ k = -6$

16. ABCD is a rectangle, find the values of x and y.



Ans. $x + y = 12 \ldots (i)$

$x - y = 8 \ldots (ii)$

on adding (i) and (ii),

$2x = 20$

$\Rightarrow x = 10$

$\therefore \ 10 + y = 12$

$\Rightarrow y = 2$

17. Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variable such that the geometrical representation of the pair so formed is

(a) intersecting lines

(b) parallel lines

(c) overlapping

Ans. 2x + 3y – 8 = 0 another linear equation representing.

(i) Intersecting lines is $x+3y=8$

(ii) Parallel lines is $4x+6y=3$

(iii) Overlapping lines is $6x+9y=24$

18. Find the value of 'k' for which the system of equation has infinitely many solutions 2x + (k – 2) y = k and 6x + (2k – 1) y = 2k + 5

Ans. for infinitely many solution

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

i.e. $\frac{2}{6}=\frac{k-2}{2k-1}=\frac{k}{2k+5}$

if $\frac{1}{3}=\frac{k-2}{2k-1}$

$2k-1=3k-6$

$k=5$

or if $\frac{k}{2k+5}=\frac{1}{3}$

or $3k=2k+5$

$k=5$

if $\frac{k-2}{2k-1}=\frac{k}{2k+5}$

$\Rightarrow 2k^2+5k-4k-10$

$=2k^2-k$

19. Find the relation between a, b, c and d for which the equations ax + by = c and cx +

dy = a have a unique solution.

Ans. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. $\frac{a}{c} \neq \frac{b}{d}$

or $ad \neq bc$

20. Solve for 'x' and 'y':

(a – b) x + (a + b) y = a^2 – b^2 – 2ab

(a + b) (x + y) = a^2 + b^2

Ans.

$$\begin{array}{l} (a-b)x+(a+b)y=a^2-b^2-2ab \\ \underline{\overset{-}{}(a+b)x \overset{-}{+}(a+b)y \overset{-}{=} a^2+b^2} \\ -2bx \qquad\qquad = -2b(b+a) \end{array}$$

$$x=a+b$$

$$\therefore\ (a-b)(a+b)+(a+b)y=a^2-b^2-2ab$$

$$a^2-b^2+(a+b)y$$

$$=a^2-b^2-2ab$$

$$y=\frac{-2ab}{a+b}$$

CBSE Class 10 Mathematics
Important Questions
Chapter 3
Pair of Linear Equations

3 Marks Questions

1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Ans. Let the present age of Aftab and his daughter be x and y respectively.

Seven years ago, Age of Aftab = x - 7 and Age of his daughter = y - 7

According to the given condition,

$$(x-7)=7(y-7)$$

$$\Rightarrow x-7=7y-49$$

$$\Rightarrow x-7y=-42$$

Again, Three years hence, Age of Aftab = x + 3 and Age of his daughter = y + 3

According to the given condition,

$$(x+3)=3(y+3)$$

$$\Rightarrow x+3=3y+9$$

$$\Rightarrow x-3y=6$$

Thus, the given conditions can be algebraically represented as:

$x - 7y = -42$

$\Rightarrow x = -42 + 7y$

Three solutions of this equation can be written in a table as follows:

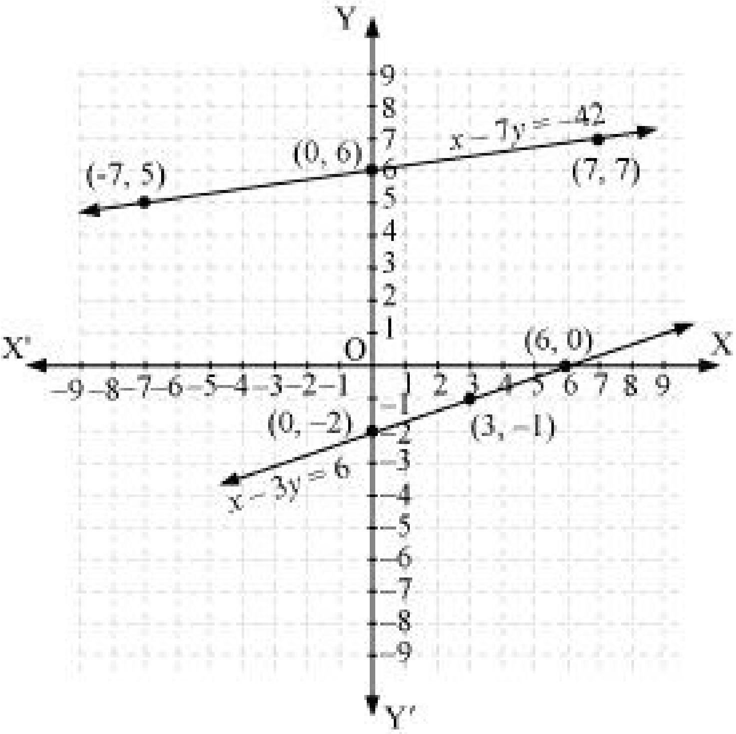
x	–7	0	7
y	5	6	7

And x – 3y = 6 $\Rightarrow$ x = 6 + 3y

Three solutions of this equation can be written in a table as follows:

x	6	3	0
y	0	–1	–2

The graphical representation is as follows:



Concept insight: In order to represent the algebraic equations graphically the solution set of equations must be taken as whole numbers only for the accuracy. Graph of the two linear equations will be represented by a straight line.

2. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of class X took part in a mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

Ans. (i) Let number of boys who took part in the quiz = x

Let number of girls who took part in the quiz = y

According to given conditions, we have

$x+y=10$ ... **(1)**

And, $y=x+4$

$\Rightarrow x-y=-4$ ... **(2)**

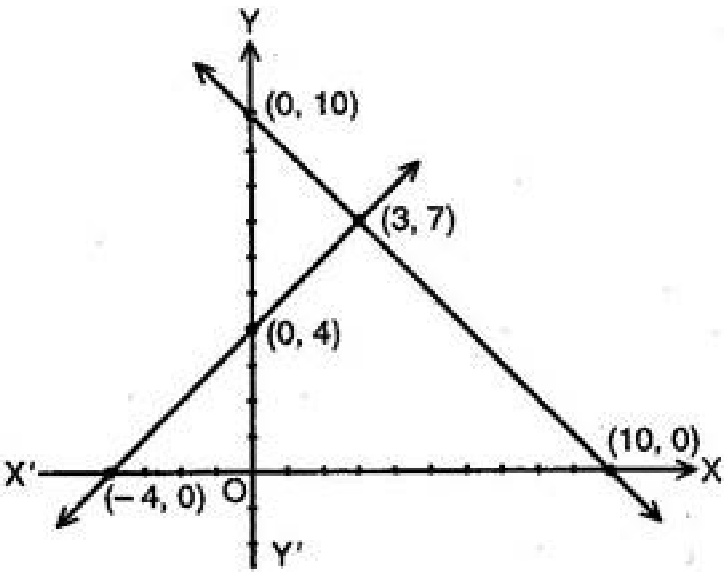
For equation x + y = 10, we have following points which lie on the line

x	0	10
y	10	0

For equation x – y = –4, we have following points which lie on the line

x	0	-4
y	4	0

We plot the points for both of the equations to find the solution.



We can clearly see that the intersection point of two lines is **(3, 7).**

Therefore, number of boys who took park in the quiz = 3 and, number of girls who took part in the quiz = 7.

(ii) Let cost of one pencil = Rs x and Let cost of one pen = Rs y

According to given conditions, we have

$5x+7y=50$ ... **(1)**

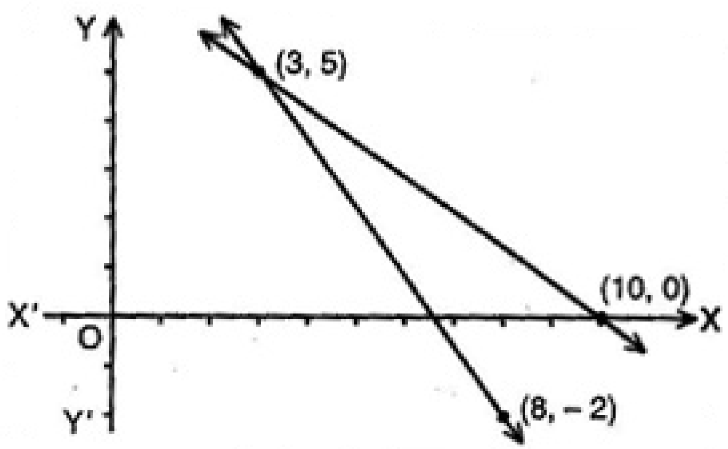
$7x+5y=46$ ... **(2)**

For equation 5x + 7y = 50, we have following points which lie on the line

x	10	3
y	0	5

For equation 7x + 5y = 46, we have following points which lie on the line

x	8	3
y	-2	5



We can clearly see that the intersection point of two lines is **(3, 5).**

Therefore, cost of pencil = Rs 3 and, cost of pen = Rs 5

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) 5x − 4y + 8 = 0

7x+ 6y – 9 = 0

(ii) 9x+ 3y + 12 = 0

18x + 6y + 24 = 0

(iii) 6x − 3y + 10 = 0

2x– y + 9 = 0

Ans. (i) $5x-4y+8=0$, $7x+6y-9=0$

Comparing equation $5x-4y+8=0$ with $a_1x+b_1y+c_1=0$ and $7x+6y-9=0$ with $a_2x+b_2y+c_2=0$,

We get $a_1=5$, $b_1=-4$, $c_1=8$, $a_2=7$, $b_2=6$, $c_2=-9$

We have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ because $\frac{5}{7} \neq \frac{-4}{6}$

Hence, lines have unique solution which means they intersect at one point.

(ii) $9x+3y+12=0$, $18x+6y+24=0$

Comparing equation $9x+3y+12=0$ with $a_1x+b_1y+c_1=0$ and $18x+6y+24=0$ with $a_2x+b_2y+c_2=0$,

We get, $a_1= 9, b_1= 3, c_1= 12$, $a_2= 18, b_2=6, c_2= 24$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ because $\frac{9}{18} = \frac{3}{6} = \frac{12}{24}$

Hence, lines are coincident.

(iii) $6x-3y+10=0$, $2x-y+9=0$

Comparing equation $6x-3y+10=0$ with $a_1x+b_1y+c_1=0$ and $2x-y+9=0$ with $a_2x+b_2y+c_2=0$,

We get, $a_1= 6, b_1= -3, c_1= 10$, $a_2= 2, b_2= -1, c_2= 9$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ because $\frac{6}{2} = \frac{-3}{-1} \neq \frac{10}{9}$

Hence, lines are parallel to each other.

4. Given the linear equation (2x+3y – 8=0), write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) Intersecting lines

(ii) Parallel lines

(iii) Coincident lines

Ans. (i) Let the second line be equal to $a_2x+b_2y+c_2=0$

Comparing given line **2x+3y – 8=0** with $a_1x+b_1y+c_1=0$,

We get$a_1=2, b_1=3$ and $c_1=-8$

Two lines intersect with each other if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, second equation can be **x+2y=3** because $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) Let the second line be equal to $a_2x+b_2y+c_2=0$

Comparing given line **2x+3y – 8=0 with** $a_1x+b_1y+c_1=0$,

We get$a_1=2, b_1=3$ and $c_1=-8$

Two lines are parallel to each other if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, second equation can be **2x+3y – 2=0** because $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) Let the second line be equal to $a_2x+b_2y+c_2=0$

Comparing given line **2x+3y – 8=0 with** $a_1x+b_1y+c_1=0$,

We get $a_1=2, b_1=3$ and $c_1=-8$

Two lines are coincident if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

So, second equation can be **4x+6y – 16=0** because $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

5. Solve $2x+3y=11$ and $2x-4y=-24$ and hence find the value of 'm' for which $y=mx+3$.

Ans. $2x+3y=11$ ... (1)

$2x-4y=-24$ ... (2)

Using equation (2), we can say that

$2x=-24+4y$

$\Rightarrow x=-12+2y$

Putting this in equation (1), we get

$2(-12+2y)+3y=11$

$\Rightarrow -24+4y+3y=11$

$\Rightarrow 7y=35$

$\Rightarrow y=5$

Putting value of y in equation (1), we get

$2x+3(5)=11$

$\Rightarrow 2x+15=11$

$\Rightarrow 2x=11-15=-4$

$\Rightarrow x=-2$

Therefore, $x=-2$ and $y=5$

Putting values of x and y in $y=mx+3$, we get

$5=m(-2)+3$

$\Rightarrow 5=-2m+3$

$\Rightarrow -2m=2$

$\Rightarrow m=-1$

6. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7$

$(a-b)x + (a+b)y = 3a + b - 2$

(ii) For which value of k will the following pair of linear equations have no solution?

$3x + y = 1$

$(2k-1)x + (k-1)y = 2k + 1$

Ans. (i) Comparing equation $2x + 3y - 7 = 0$ with $a_1x + b_1y + c_1 = 0$ and $(a-b)x + (a+b)$

$y - 3a - b + 2 = 0$ with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 2$, $b_1=3$ and $c_1 = -7$, $a_2 = (a-b)$, $b_2 = (a+b)$ and $c_2 = 2 - b - 3a$

Linear equations have infinite many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{2-b-3a}$

$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} \text{ and } \frac{3}{a+b} = \frac{-7}{2-b-3a}$

$\Rightarrow 2a + 2b = 3a - 3b$ and $6 - 3b - 9a = -7a - 7b$

$\Rightarrow a = 5b$ ... (1) and $-2a = -4b - 6$ ... (2)

Putting (1) in (2), we get

$-2(5b) = -4b - 6$

$\Rightarrow -10b + 4b = -6$

$\Rightarrow -6b = -6 \quad \Rightarrow b = 1$

Putting value of b in (1), we get

$a = 5b = 5(1) = 5$

Therefore, $a = 5$ and $b = 1$

(ii) Comparing $(3x + y - 1 = 0)$ with $a_1x + b_1y + c_1 = 0$ and $(2k - 1)x + (k - 1)y - 2k - 1 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 3, b_1 = 1$ and $c_1 = -1$, $a_2 = (2k - 1)$, $b_2 = (k - 1)$ and $c_2 = -2k - 1$

Linear equations have no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-2k-1}$

$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1}$

$\Rightarrow 3(k - 1) = 2k - 1$

$\Rightarrow 3k - 3 = 2k - 1$

$\Rightarrow k = 2$

7. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h, it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Ans. Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel be d km.

Since Speed = $\dfrac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$

$\Rightarrow x = \dfrac{d}{t}$

$\Rightarrow d = xt$ ... (1)

According to the question

$x + 10 = \dfrac{d}{t - 2}$

$\Rightarrow (x+10)(t-2) = d$

$\Rightarrow xt + 10t - 2x - 20 = d$

$\Rightarrow -2x + 10t = 20$ ......(2) [Using eq. (1)]

Again, $x - 10 = \dfrac{d}{t+3}$

$\Rightarrow (x-10)(t+3) = d$

$\Rightarrow xt - 10t + 3x - 30 = d$

$\Rightarrow 3x - 10t = 30$ ......(3) [Using eq. (1)]

Adding equations (2) and (3), we obtain:

x = 50

Substituting the value of x in equation (2), we obtain:

(–2) x (50) + 10t = 20

$\Rightarrow$ –100 + 10t = 20

$\Rightarrow$ 10t = 120   t = 12

From equation (1), we obtain:

d = xt = 50 x 12 = 600

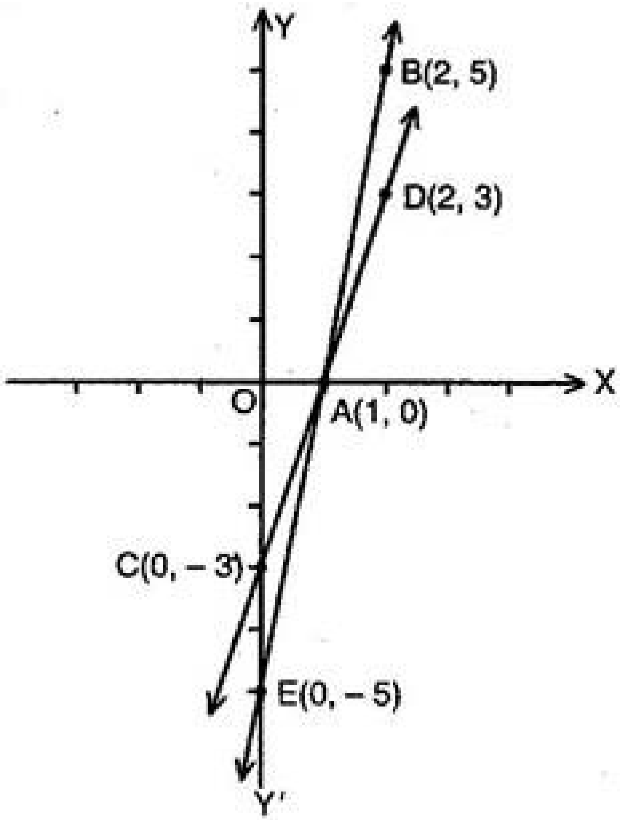
Thus, the distance covered by the train is 600 km.

8. Draw the graphs of the equations $5x-y=5$ and $3x-y=3$. Determine the co-ordinate of the vertices of the triangle formed by these lines and the $y-$ axis.

Ans. $5x-y=5$

$\Rightarrow y=5x-5$

Three solutions of this equation can be written in a table as follows:



x	0	1	2
y	–5	0	5

$3x - y = 3$

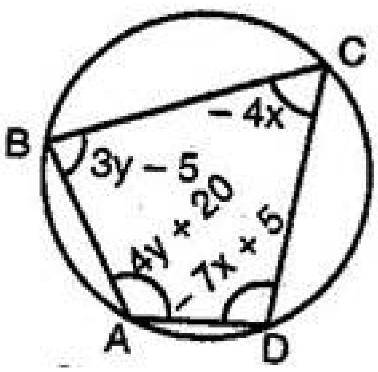
$\Rightarrow y = 3x - 3$

x	0	1	2
y	–3	0	3

It can be observed that the required triangle is ΔABC.

The coordinates of its vertices are A (1, 0), B (0, –3), C (0, –5).

9. ABCD is a cyclic quadrilateral (see figure). Find the angles of the cyclic quadrilateral.



Ans. We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°.

$\therefore \angle A + \angle C = 180°$

$\Rightarrow 4y + 20 - 4x = 180°$

$\Rightarrow -4x + 4y = 160°$

$\Rightarrow x - y = -40°$ ....(1)

Also $\angle B + \angle D = 180°$

$\Rightarrow 3y - 5 - 7x + 5 = 180°$

$\Rightarrow -7x + 3y = 180°$ ....(2)

Multiplying equation (1) by 3, we obtain:

$3x - 3y = -120°$ ....(3)

Adding equations (2) and (3), we obtain:

$-4x = 60°$

$\Rightarrow x = -15°$

Substituting the value of x in equation (1), we obtain:

$-15 - y = -40°$

$\Rightarrow y = -15 + 40 = 25$

$\therefore \angle A = 4y + 20 = 4 \times 25 + 20 = 120°$

$\angle B = 3y - 5 = 3 \times 25 - 5 = 70°$

$\angle C = -4x = -4 \times (-15) = 60°$

$\angle D = -7x + 5 = -7(-15) + 5 = 110°$

10. Draw graphs of the equations on the same graph paper 2x + 3y = 12, x – y = 1. Find the area and co-ordinate of the vertices of the triangle formed by the two straight lines and the y-axis.

Ans. $2x + 3y = 12$

$x - y = 1$

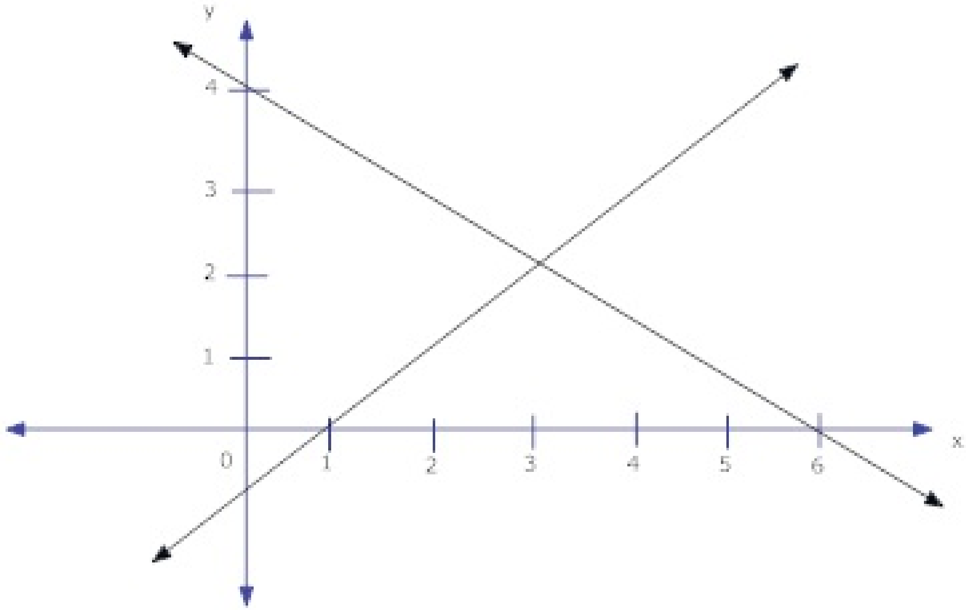
$when\ x = \dfrac{12 - 3y}{2}$

x	6	0
y	0	4

$when\ x = y + 1$

x	1	2

y	0	1



$$A = \frac{1}{2} b \times h$$

$$= \frac{1}{2} \times 5 \times 2 = 5 \text{ square units}$$

11. Solve: $\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}$ **and** $\frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$

Ans. Let $\frac{1}{3x+2y} = u$

and $\frac{1}{3x-2y} = v$

$\Rightarrow \ 2u + 3v = \frac{17}{5}$

and $5u + v = 2$

on solving, we get $u = \frac{1}{5}$

and $v = 1$

$\therefore\ 3x+2y=5$

$3x-2y=1$

on solving, we get

$x=1$ and $y=1$

12. The sum of a two-digit number and the number obtained by reversing the order of digits is 99. If the digits differ by 3, find the number.

Ans. Let the digit at unit place be 'x' and tens place be 'y'.

According to question,

$(10y+x)+(10x+y)=99$

or $x+y=9 \rightarrow (i)$

and $x-y=3 \rightarrow (ii)$

or $y-x=3 \rightarrow (iii)$

on solving eq (i) and (ii), we get

$x=6$ and $y=3$

then the original number is 36.

on solving eq (i) and (iii), we get,

$x=3$ and $y=6$

$\therefore$ The number is '63'

13. In a cyclic quadrilateral ABCD, $\angle A=(2x+4)^\circ$, $\angle B=(y+3)^\circ$, $\angle C=(2y+10)^\circ$ and $\angle D=(4x-5)^\circ$ Find the four angles.

Ans. In cyclic quadrilateral

$\angle A+\angle C=180$ and $\angle B+\angle D=180$

$\Rightarrow 2x+4+2y+10=180$

$\Rightarrow x+y=83 \rightarrow (i)$

and $\angle B+\angle D=180$

$\Rightarrow y+3+4x-5=180$

$\Rightarrow 4x+y=182 \rightarrow (ii)$

on solving eq (i) and (ii), we get,

$x=33$ and $y=50$

$\therefore$ Angles are

$\angle A=2x+4=2\times 33+4=70^{\circ}$

$\angle B=y+3=50+3=53^{\circ}$

$\angle C=2y+10=2\times 50+10=110^{\circ}$

$\angle D=4x-5=4\times 33-5=127^{\circ}$

14. A two-digit number is obtained by either multiplying the sum of the digits by 8 and adding 1 or by multiplying number. How many such numbers are there?

Ans. Let digit at unit place x and tens place y then

original number $=(10y+x)$

According to question,

$10y+x=8(x+y)+1$

or $7x-2y+1=0 \rightarrow (i)$

or $13(x-y)+2=10y+x$

or $12x-23y+2=0 \rightarrow (ii)$

on solving eq (i) and (ii), we get,

$y=\dfrac{2}{16y}$

which is not possible

$\therefore\ 13(y-3)+2=10y+x$

or $14x-3y=2 \rightarrow (iii)$

on solving eq (i) and (ii) we get,

$x=1,\ y=4$

$\therefore$ original number $=41$

only one number exist.

15. A leading library has a fixed charge for the first three days and an additional charge for each day thereafter Sarika paid Rs. 27 for a book kept for seven days, while Sury paid Rs. 21 for the book she kept for five days, find the fixed charge and the charge for each extra day.

Ans. Let the fixed charge be Rs x and additional charge be Rs y.

According to question,

$x+(7-3)y=27$

or $x+4y=27 \rightarrow (i)$

and $x+(5-3)y=21$

$x+2y=21 \rightarrow (ii)$

on solving eq (i) and (ii), we get

$x=15,\ y=3$

16. If 2 is added to the numerator of a fraction, it reduces to $\frac{1}{2}$ and if 1 is subtracted from the denominator, it reduces to $\frac{1}{3}$. Find the fraction.

Ans. Let the fraction be $\frac{x}{y}$

According to question,

$$\frac{x+2}{y}=\frac{1}{2}$$

or $2x-y=-4 \rightarrow (i)$

and $\frac{x}{y-1}=\frac{1}{3}$

or $3x-y=-1 \rightarrow (ii)$

on solving eq (i) and (ii), we get,

$x=3,\ y=10$

$\therefore$ fraction is $\frac{3}{10}$

17. Solve the following system of equation graphically.

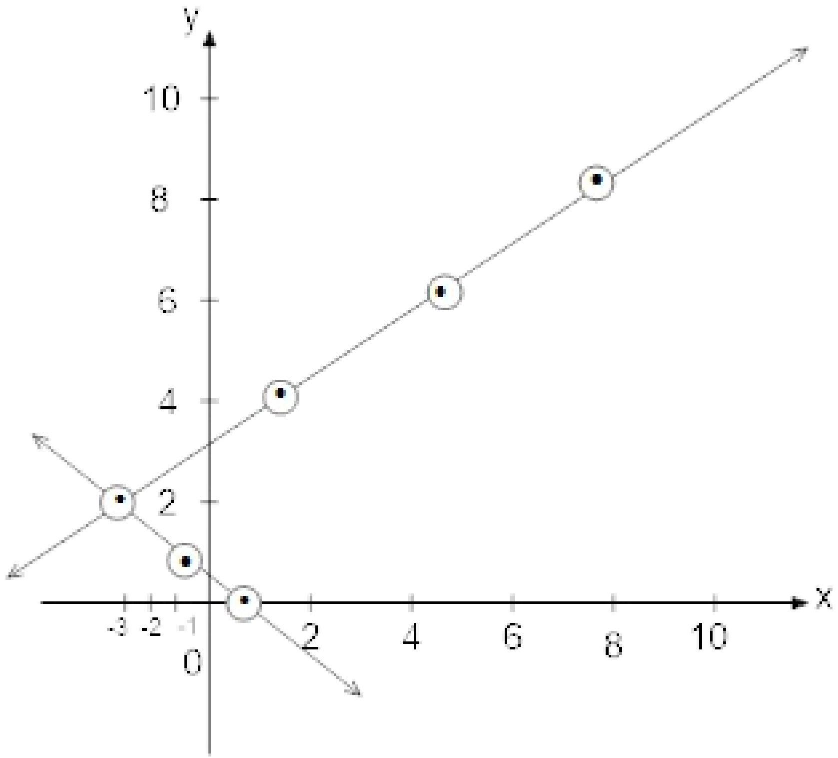
x + 2y = 1, x – 2y = -7, also read the points from the graph where the lines meet the x-axis and y-axis.

Ans. $x+2y=1 \rightarrow (i)$

x	0	1	-1
y	1/2	0	1

$x-2y=-7 \rightarrow (ii)$

x	1	5	9
y	4	6	8



$x = -3$
$y = 2$

The straight line (i) meet the axis at $\left(0, \frac{1}{2}\right)$ and $(1, 0)$

and straight line (ii) meet the axis at $\left(0, \frac{7}{2}\right)$ and $(-7, 0)$.

18. Solve 23x – 29y = 98 and 29x – 23y = 110.

Ans. $23x - 29y = 98 \rightarrow (i)$

$29x - 23y = 110 \rightarrow (ii)$

on adding eq (i) and (ii)

$52x - 52y = 208$

$or\ x - y = 4 \rightarrow (iii)$

on subtracting

$$\begin{array}{r} 23x - 29y = 98 \\ 29x - 23y = 110 \\ \hline -6x - 6y = -12 \end{array}$$

$x + y = 2 \rightarrow (iv)$

on adding (iii) and (iv) we get

$2x = 6$ i.e. $x = 3$

$\therefore\ 3 + y = 2$

$y = -1$

$\therefore\ x = 3$ and $y = -1$

19. A man has only 20 paise coins and 25 paisea coins in his purse. If he has 50 coins in all totaling Rs 11.25. How many coins of each kind does he have?

Ans. Let the number of coin of

20 paise be 'x' and

25 paise be 'y'

According to question

$x + y = 50 \rightarrow (i)$

and $20x + 25y = 1125 \rightarrow (ii)$

or

$$\begin{array}{r} 4x + 5y = 225 \\ 4x + 4y = 200 \quad [\text{from } (i)] \\ \hline y = 25 \end{array}$$

$x + y = 50$

$\Rightarrow x + 25 = 50 = 25$

$\therefore$ Number of 20 paise coin = 25

and number of 25 paise coin = 25

20. A says to B my present age is five times your that age when I was an old as you are now. It the sum of their present ages is 48 years, find their present ages.

Ans. Let the present age of A = x years

and B = y years

According to question,

$x+y=48 \rightarrow (i)$

$x=5[y-(x-y)]$

$x=5[2y-x]$

$x=10y-5x$

$3x=5y$

$3(48-y)=5y$

$\Rightarrow y=18$ years

and $x=48-18$ years

$x=30$ years

21. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours it can go 40 km upstream and 55 km down stream. Determine the speed of the stream and that of the boat in still water.

Ans. Let the speed of boat is x km/h in still water

and stream y km/h

According to question,

$$\frac{30}{x-y}+\frac{44}{x+y}=10$$

and $\dfrac{40}{x-y}+\dfrac{55}{x+y}=13$

Let $\dfrac{1}{x-y}=u$ and $\dfrac{1}{x+y}=v$

$30u+44v=10 \rightarrow (i)$

$40u+55v=13 \rightarrow (ii)$

on solving eq (i) and (ii) we get,

$u=\dfrac{1}{5}\Rightarrow x-y=5 \rightarrow (iii)$

$v=\dfrac{1}{11}\Rightarrow x+y=11 \rightarrow (iv)$

on solving eq (iii) and (iv) we get,

$x=8km/h$

$y=3km/h$

22. Determine graphically the coordinates of the vertices of the triangle, the equation of whose sides are y = x, 3y = x and x + y = 8.

Ans. *when* $y=x$

X	1	2
Y	1	2

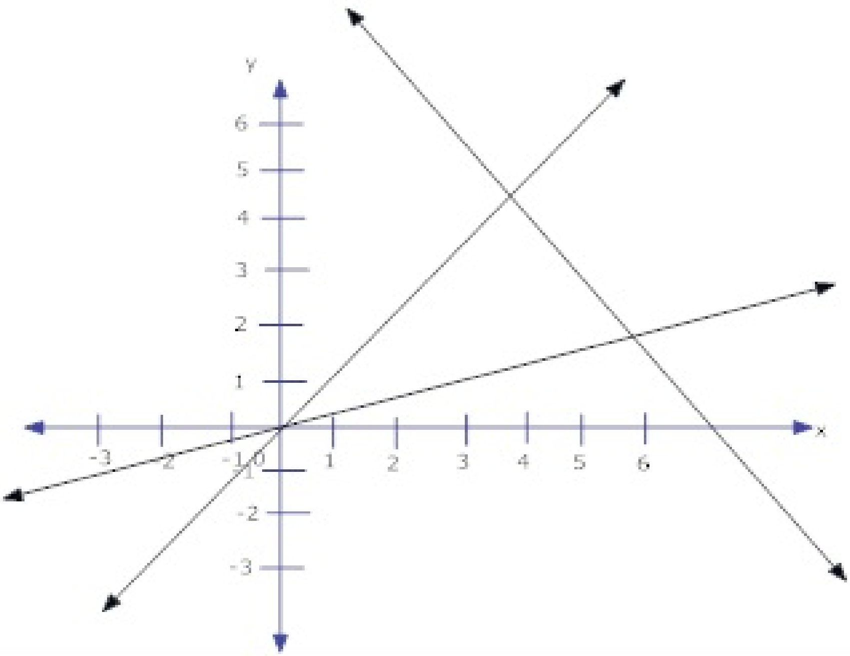
when $3y=x$

X	6	3
Y	2	1

when $x+y=8$ or $y=8-x$

X	4	5

Y	4	3



23. Father's age is three times the sum of ages of his two children. After 5 years, his age will be twice the sum of ages of two children. Find the age of father.

Ans.

Let the present age of father be x years and sum of present age of two son's be y years.

According to question,

after five years

$x+5=2(y+5+5)$

$x+5=2y+20$

$x-2y=15 \rightarrow (i)$

and $x=3y \rightarrow (ii)$

$\therefore\ 3y-2y=15$

or $y=15$

$\therefore$ age of father $x=3y$

$=3\times 15$

$=45$ years

24. On selling a T.V. at 5% gain and a fridge at 10% gain shop keeper gains Rs 2000. But if he sells the T.V at 10% gain and the Fridge at 5% loss, he gains Rs 1500 on the transaction. Find the actual Price of TV and Fridge.

Ans. Let the selling price of TV = Rs x

and fridge = Rs y

According to question,

5% of x + 10% of $y = 2000$

or $\frac{x}{20}+\frac{y}{10}=2000$

$x+2y=40000 \rightarrow (i)$

and 10% of x + 5% of $y = 1500$

or $\frac{x}{10}+\frac{y}{20}=1500$

$2x+y=30000 \rightarrow (ii)$

on solving eq (i) and (ii), we get,

$x=\text{Rs } \frac{20000}{3};\ y=\text{Rs } \frac{50000}{3}$

25. A taken 3 hours more than B to walk a distance of 30 km. But if A doubles his speed, he is ahead of B by $1\frac{1}{2}$ hours. Find their original speed.

Ans. Let the original speed of A and B are x km/h and y km/h respectively.

According to question,

$\frac{30}{x}-\frac{30}{y}=3$

or $\frac{1}{x}-\frac{1}{y}=\frac{1}{10}$

or $u-v=\frac{1}{10} \rightarrow (i)$

$$and\ \frac{30}{y}-\frac{30}{2x}=\frac{3}{2}$$

$$\frac{1}{y}-\frac{1}{2x}=\frac{1}{20}$$

$$v-\frac{u}{2}=\frac{1}{20}\ \rightarrow\ (ii)$$

on adding (i) and (ii), we get,

$$\frac{4}{2}=\frac{3}{20}\Rightarrow\frac{1}{u}=\frac{10}{3}$$

$$\text{or}\ \ x=\frac{10}{3}$$

$$and\ \ y=5$$

$\therefore$ speed of A is 3.3 km/h

CBSE Class 10 Mathematics

Important Questions

Chapter 3

Pair of Linear Equations

4 Marks Questions

1. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $x+y=5,\ 2x+2y=10$

(ii) $x-y=8,\ 3x-3y=16$

(iii) $2x+y=6,\ 4x-2y=4$

(iv) $2x-2y-2=0,\ 4x-4y-5=0$

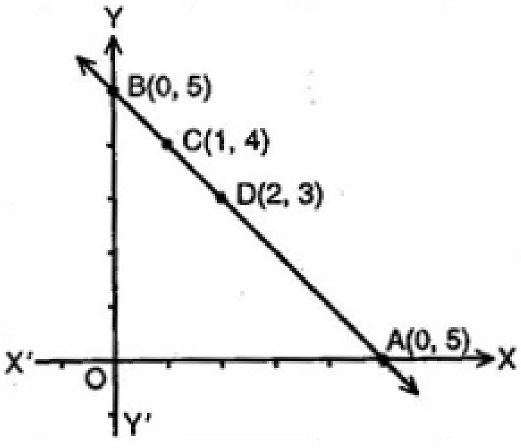
Ans. (i) $x+y=5,\ 2x+2y=10$

For equation x + y – 5 = 0, we have following points which lie on the line

x	0	5
y	5	0

For equation 2x + 2y – 10 = 0, we have following points which lie on the line

x	1	2
y	4	3



We can see that both of the lines coincide. Hence, there are infinite many solutions. Any point which lies on one line also lies on the other. Hence, by using equation ($x+y-5=0$), we can say that$x=5-y$

We can assume any random values for y and can find the corresponding value of x using the above equation. All such points will lie on both lines and there will be infinite number of such points.

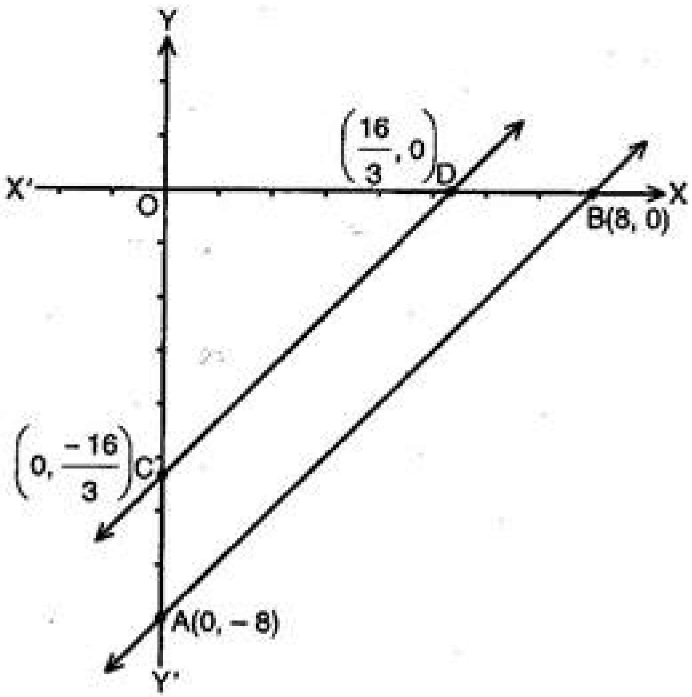
(ii) $x-y=8$, $3x-3y=16$

For x – y = 8, the coordinates are:

x	0	8
y	-8	0

And for 3x – 3y = 16, the coordinates

x	0	$\frac{16}{3}$
y	$\frac{-16}{3}$	0



Plotting these points on the graph,it is clear that both lines are parallel. So the two lines have no common point. Hence the given equations have no solution and lines are inconsistent.

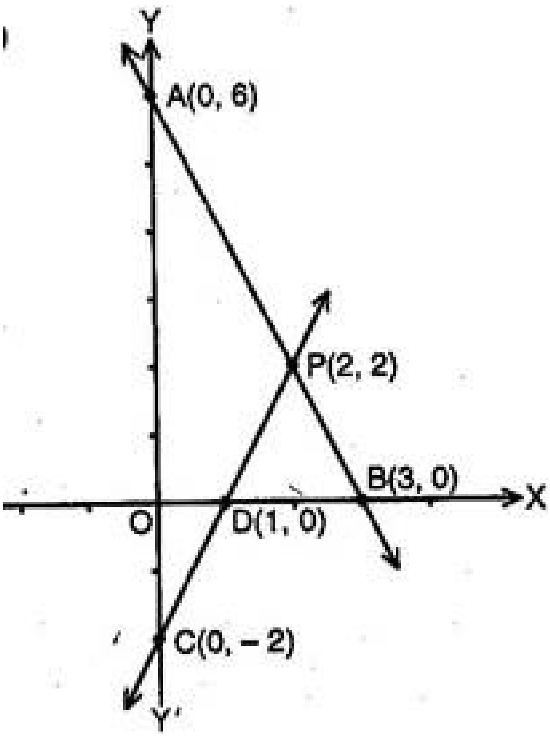
(iii) $2x + y = 6$, $4x - 2y = 4$

For equation 2x + y – 6 = 0, we have following points which lie on the line

x	0	3
y	6	0

For equation 4x – 2y – 4 = 0, we have following points which lie on the line

x	0	1
y	-2	0



We can clearly see that lines are intersecting at (2, 2) which is the solution.

Hence x = 2 and y = 2 and lines are consistent.

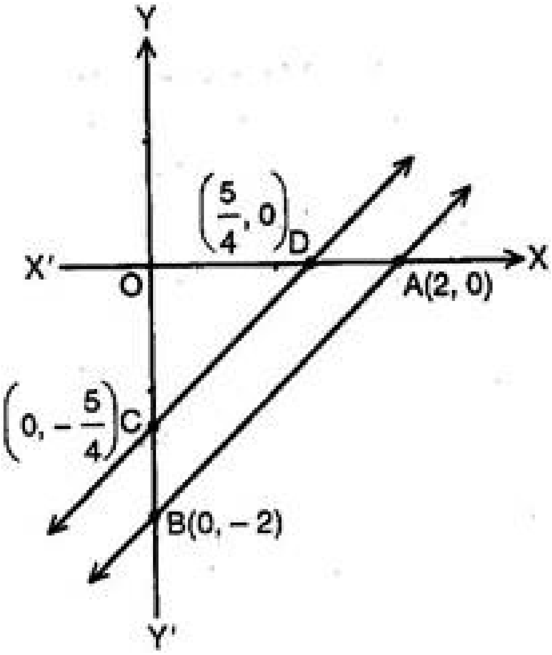
(iv) $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$

For 2x – 2y – 2 = 0, the coordinates are:

x	2	0
y	0	-2

And for 4x – 4y – 5 = 0, the coordinates

x	0	$\frac{5}{4}$
y	$\frac{-5}{4}$	0



Plotting these points on the graph, it is clear that both lines are parallel. So the two lines have no common point. Hence the given equations have no solution and lines are inconsistent.

2. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$

$x - y = 4$

(ii) $s - t = 3$

$$\frac{s}{3} + \frac{t}{2} = 6$$

(iii) $3x - y = 3$

$9x - 3y = 9$

(iv) ***0.2x + 0.3y = 1.3***

0.4x + 0.5y = 2.3

(v) $\sqrt{2}x+\sqrt{3}y=0$

$\sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

Ans. (i) $x+y=14$ … (1)

$x-y=4$ … (2)

$x=4+y$ from equation (2)

Putting this in equation (1), we get

$4+y+y=14$

$\Rightarrow 2y=10$

$\Rightarrow y=5$

Putting value of y in equation (1), we get

$x+5=14$

$\Rightarrow x=14-5=9$

Therefore, $x=9$ and $y=5$

(ii) $s-t=3$ … (1)

$\frac{s}{3}+\frac{t}{2}=6$ … (2)

Using equation (1), we can say that $s=3+t$

Putting this in equation (2), we get

$$\frac{3+t}{3}+\frac{t}{2}=6$$

$$\Rightarrow \frac{6+2t+3}{6}=6$$

$\Rightarrow 5t+6=36$

$\Rightarrow 5t=30$

$\Rightarrow t=6$

Putting value of t in equation (1), we get

$s-6=3$

$\Rightarrow s=3+6=9$

Therefore, $t=6$ and $s=9$

(iii) $3x-y=3$ ... (1)

$9x-3y=9$ ... (2)

Comparing equation $3x-y=3$ with $a_1x+b_1y+c_1=0$ and equation $9x-3y=9$ with $a_2x+b_2y+c_2=0$,

We get $a_1=3, b_1=-1$, $c_1=-3$, $a_2=9, b_2=-3$ and $c_2=-9$

Here $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Therefore, we have infinite many solutions for x and y

(iv) $0.2x+0.3y=1.3$ ... (1)

$0.4x+0.5y=2.3$ ... (2)

Using equation (1), we can say that

$0.2x = 1.3 - 0.3y$

$\Rightarrow x = \dfrac{1.3 - 0.3y}{0.2}$

Putting this in equation (2), we get

$0.4\left(\dfrac{1.3 - 0.3y}{0.2}\right) + 0.5y = 2.3$

$\Rightarrow 2.6 - 0.6y + 0.5y = 2.3$

$\Rightarrow -0.1y = -0.3$

$\Rightarrow y = 3$

Putting value of y in (1), we get

$0.2x + 0.3(3) = 1.3$

$\Rightarrow 0.2x + 0.9 = 1.3$

$\Rightarrow 0.2x = 0.4$

$\Rightarrow x = 2$

Therefore, $x = 2$ and $y = 3$

(v) $\sqrt{2}x + \sqrt{3}y = 0$ ....(1)

$\sqrt{3}x - \sqrt{8}y = 0$ ....(2)

Using equation (1), we can say that

$x = \dfrac{-\sqrt{3}y}{\sqrt{2}}$

Putting this in equation (2), we get

$$\sqrt{3}\left(\frac{-\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0$$

$$\Rightarrow \frac{-3y}{\sqrt{2}}-\sqrt{8}y=0$$

$$\Rightarrow y\left(\frac{-3}{\sqrt{2}}-\sqrt{8}\right)=0$$

$\Rightarrow y=0$

Putting value of y in (1), we get $x=0$

Therefore, $x=0$ and $y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$ ... (1)

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$ ... (2)

Using equation (2), we can say that

$$x=\left(\frac{13}{6}-\frac{y}{2}\right)\times 3$$

$$\Rightarrow x=\frac{13}{2}-\frac{3y}{2}$$

Putting this in equation (1), we get

$$\frac{3}{2}\left(\frac{13}{2}-\frac{3y}{2}\right)-\frac{5y}{3}=\frac{-2}{1}$$

$$\Rightarrow \frac{39}{4}-\frac{9y}{4}-\frac{5y}{3}=-2$$

$$\Rightarrow \frac{-27y-20y}{12} = -2-\frac{39}{4}$$

$$\Rightarrow \frac{-47y}{12} = \frac{-8-39}{4}$$

$$\Rightarrow \frac{-47y}{12} = \frac{-47}{4}$$

$\Rightarrow y=3$

Putting value of y in equation (2), we get

$$\frac{x}{3}+\frac{3}{2}=\frac{13}{6}$$

$$\Rightarrow \frac{x}{3}=\frac{13}{6}-\frac{3}{2}=\frac{13-9}{6}=\frac{4}{6}=\frac{2}{3}$$

$$\Rightarrow \frac{x}{3}=\frac{2}{3}$$

$\Rightarrow x = 2$

Therefore, $x=2$ and $y=3$

3. Form a pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the

distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Ans. (i) Let first number be x and second number be y.

According to given conditions, we have

$x - y = 26$ (assuming x>y) … (1)

$x = 3y$ (x>y) … (2)

Putting equation (2) in (1), we get

$3y - y = 26 \Rightarrow 2y = 26 \Rightarrow y = 13$

Putting value of y in equation (2), we get

$x = 3y = 3 \times 13 = 39$

Therefore, two numbers are 13 and 39.

(ii) Let smaller angle $= x$and let larger angle $= y$

According to given conditions, we have

$y = x + 18$ … (1)

Also, $x + y = 180^0$ (Sum of supplementary angles) … (2)

Putting (1) in equation (2), we get

$x+x+18=180$

$\Rightarrow 2x=180-18=162$

$\Rightarrow x=81^0$

Putting value of x in equation (1), we get

$y=x+18=81+18=99^0$

Therefore, two angles are 81^0 and 99^0.

(iii) Let cost of each bat = Rs x and let cost of each ball = Rs y

According to given conditions, we have

$7x+6y=3800$ ... (1)

And, $3x+5y=1750$ ... (2)

Using equation (1), we can say that

$7x=3800-6$

$\Rightarrow x=\frac{3800-6y}{7}$

Putting this in equation (2), we get

$3\left(\frac{3800-6y}{7}\right)+5y=1750$

$\Rightarrow \left(\frac{11400-18y}{7}\right)+5y=1750$

$\Rightarrow \frac{5y}{1}-\frac{18y}{7}=\frac{1750}{1}-\frac{11400}{7}$

$\Rightarrow \frac{35y-18y}{7}=\frac{12250-11400}{7}$

$\Rightarrow 17y=850$

$\Rightarrow y=50$

Putting value of y in (2), we get

$3x+250=1750$

$\Rightarrow 3x=1500$

$\Rightarrow x=500$

Therefore, cost of each bat = Rs 500 and cost of each ball = Rs 50

(iv) Let fixed charge = Rs x and let charge for every km = Rs y

According to given conditions, we have

$x+10y=105$ … (1)

$x+15y=155$ … (2)

Using equation (1), we can say that

$x=105-10y$

Putting this in equation (2), we get

$105-10y+15y=155$

$\Rightarrow 5y=50$

$\Rightarrow y=10$

Putting value of y in equation (1), we get

$x+10(10)=105$

$\Rightarrow x=105-100=5$

Therefore, fixed charge = Rs 5 and charge per km = Rs 10

To travel distance of 25 Km, person will have to pay = Rs (x+25y) = Rs (5+25 × 10) = Rs(5+250) = Rs 255

(v) Let numerator = x and let denominator = y

According to given conditions, we have

$$\frac{x+2}{y+2}=\frac{9}{11} \quad \dots (1)$$

$$\frac{x+3}{y+3}=\frac{5}{6} \quad \dots (2)$$

Using equation (1), we can say that

$11(x+2)=9y+18$

$\Rightarrow 11x+22=9y+18$

$\Rightarrow 11x=9y-4$

$\Rightarrow x=\frac{9y-4}{11}$

Putting value of x in equation (2), we get

$6\left(\frac{9y-4}{11}+3\right)=5(y+3)$

$\Rightarrow \frac{54y}{11}-\frac{24}{11}+18=5y+15$

$\Rightarrow -\frac{24}{11}+\frac{3}{1}=\frac{5y}{1}-\frac{54y}{11}$

$\Rightarrow -\frac{24+33}{11}=\frac{55y-54y}{11}$

$\Rightarrow y=9$

Putting value of y in (1), we get

$\frac{x+2}{9+2}=\frac{9}{11}$ ⇒ $x+2=9$ ⇒ $x=7$

Therefore, fraction = $\frac{x}{y}=\frac{7}{9}$

(vi) Let present age of Jacob = x years

Let present age of Jacob's son = y years

According to given conditions, we have

$(x+5)=3(y+5)$ … (1)

And, $(x-5)=7(y-5)$ … (2)

From equation (1), we can say that

$x+5=3y+15$

⇒ $x=10+3y$

Putting value of x in equation (2) we get

$10+3y-5=7y-35$

⇒ $-4y=-40$

⇒ $y=10$ years

Putting value of y in equation (1), we get

$x+5=3(10+5)=3\times15=45$

⇒ $x=45-5=40$ years

Therefore, present age of Jacob = 40 years and, present age of Jacob's son = 10 years

4. Solve the following pair of linear equations by the elimination method and the

substitution method:

(i) x + y = 5, 2x - 3y = 4

(ii) 3x + 4y = 10, 2x - 2y = 2

(iii) 3x − 5y – 4 = 0, 9x = 2y + 7

(iv) $\frac{x}{2}+\frac{2y}{3}=-1, x-\frac{y}{3}=3$

Ans. (i) x+y=5 ... (1)

2x – 3y=4 ... (2)

Elimination method:

Multiplying equation (1) by 2, we get equation (3)

$2x+2y=10$ ... (3)

$2x-3y=4$ ... (2)

Subtracting equation (2) from (3), we get

$5y=6$

$\Rightarrow y=\frac{6}{5}$

Putting value of y in (1), we get

$x+\frac{6}{5}=5 \Rightarrow x=5-\frac{6}{5}=\frac{19}{5}$

Therefore, $x=\frac{19}{5}$ and $y=\frac{6}{5}$

Substitution method:

$x+y=5$ ... (1)

$2x - 3y = 4$ ... (2)

From equation (1), we get,

$x = 5 - y$

Putting this in equation (2), we get

$2(5-y) - 3y = 4 \Rightarrow 10 - 2y - 3y = 4$

$\Rightarrow 5y = 6$

$\Rightarrow y = \frac{6}{5}$

Putting value of y in (1), we get

$x = 5 - \frac{6}{5} = \frac{19}{5}$

Therefore, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

(ii) $3x + 4y = 10$ ... (1)

$2x - 2y = 2$ ... (2)

Elimination method:

Multiplying equation (2) by 2, we get (3)

$4x - 4y = 4$ ... (3)

$3x + 4y = 10$ ... (1)

Adding (3) and (1), we get

$7x = 14$

$\Rightarrow x = 2$

Putting value of x in (1), we get

$3(2)+4y=10$

$\Rightarrow 4y=10-6=4$

$\Rightarrow y=1$

Therefore, $x=2$ and $y=1$

Substitution method:

$3x+4y=10$ ... (1)

$2x-2y=2$ ... (2)

From equation (2), we get

$2x=2+2y$

$\Rightarrow x=1+y$ ... (3)

Putting this in equation (1), we get

$3(1+y)+4y=10$

$\Rightarrow 3+3y+4y=10$

$\Rightarrow 7y=7$

$\Rightarrow y=1$

Putting value of y in (3), we get$x=1+1=2$

Therefore, $x=2$ and $y=1$

(iii) $3x-5y-4=0$ ... (1)

$9x=2y+7$ ... (2)

Elimination method:

Multiplying (1) by 3, we get (3)

$9x-15y-12=0$ ... (3)

$9x-2y-7=0$ ... (2)

Subtracting (2) from (3), we get

$-13y-5=0$

$\Rightarrow -13y=5$

$\Rightarrow y=\frac{-5}{13}$

Putting value of y in (1), we get

$3x-5\left(\frac{-5}{13}\right)-4=0$

$\Rightarrow 3x=4-\frac{25}{13}=\frac{52-25}{13}=\frac{27}{13}$

$\Rightarrow x=\frac{27}{13\times 3}=\frac{9}{13}$

Therefore, $x=\frac{9}{13}$ and $y=\frac{-5}{13}$

Substitution Method:

$3x-5y-4=0$ ... (1)

$9x=2y+7$ ... (2)

From equation (1), we can say that

$3x=4+5y$

$\Rightarrow x = \frac{4+5y}{3}$

Putting this in equation (2), we get

$9\left(\frac{4+5y}{3}\right) - 2y = 7$

$\Rightarrow 12 + 15y - 2y = 7$

$\Rightarrow 13y = -5$

$\Rightarrow y = \frac{-5}{13}$

Putting value of y in (1), we get

$3x - 5\left(\frac{-5}{13}\right) = 4$

$\Rightarrow 3x = 4 - \frac{25}{13} = \frac{52-25}{13} = \frac{27}{13}$

$\Rightarrow x = \frac{27}{13\times 3} = \frac{9}{13}$

Therefore, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ ...(1)

$x - \frac{y}{3} = 3$ ... (2)

Elimination method:

Multiplying equation (2) by 2, we get (3)

$2x-\frac{2}{3}y=6$ … (3)

$\frac{x}{2}+\frac{2y}{3}=-1$ … (1)

Adding (3) and (1), we get

$\frac{5}{2}x=5$

⇒ x=2

Putting value of x in (2), we get

$2-\frac{y}{3}=3$

⇒ y=−3

Therefore, x=2 and y=−3

Substitution method:

$\frac{x}{2}+\frac{2y}{3}=-1$ … (1)

$x-\frac{y}{3}=3$ … (2)

From equation (2), we can say that $x=3+\frac{y}{3}=\frac{9+y}{3}$

Putting this in equation (1), we get

$\frac{9+y}{6}+\frac{2}{3}y=-1$

⇒ $\frac{9+y+4y}{6}=-1$

$\Rightarrow 5y+9=-6$

$\Rightarrow 5y=-15$

$\Rightarrow y=-3$

Putting value of y in (1), we get

$$\frac{x}{2}+\frac{2}{3}(-3)=-1$$

$\Rightarrow x=2$

Therefore, $x=2$ and $y=-3$

5. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes ½ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as sonu. Ten years later, Nuri will be twice as old as sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Ans. (i) Let numerator $=x$and let denominator $=y$

According to given condition, we have

$$\frac{x+1}{y-1}=1 \; and \; \frac{x}{y+1}=\frac{1}{2}$$

⇒ $x+1=y-1$ and $2x=y+1$

⇒ $x-y=-2$ … **(1)** and $2x-y=1$ … **(2)**

So, we have equations **(1)** and **(2)**, multiplying equation **(1)** by 2 we get **(3)**

$2x-2y=-4$ … **(3)**

$2x-y=1$ … **(2)**

Subtracting equation **(2)** from **(3)**, we get

$-y=-5$

⇒ $y=5$

Putting value of y in **(1)**, we get

$x-5=-2$

⇒ $x=-2+5=3$

Therefore, fraction = $\frac{x}{y}=\frac{3}{5}$

(ii) Let present age of Nuri $=x$ years and let present age of Sonu $=y$ years

5 years ago, age of Nuri = (x – 5) years

5 years ago, age of Sonu = (y – 5) years

According to given condition, we have

$(x-5)=3(y-5)$

⇒ $x-5=3y-15$

⇒ $x-3y=-10$ … **(1)**

10 years later from present, age of Nuri $=(x+10)$ years

10 years later from present, age of Sonu $=(y+10)$ years

According to given condition, we have

$(x+10)=2(y+10)$

$\Rightarrow x+10=2y+20$

$\Rightarrow x-2y=10$ ... **(2)**

Subtracting equation **(1)** from **(2)**, we get

$y=10-(-10)=20$ years

Putting value of **y** in **(1)**, we get

$x-3(20)=-10$

$\Rightarrow x-60=-10$

$\Rightarrow x=50$ years

Therefore, present age of Nuri $=50$ years and present age of Sonu $=20$ years

(iii) Let digit at ten's place $=x$and Let digit at one's place $=y$

According to given condition, we have

$x+y=9$ ... **(1)**

And $9(10x+y)=2(10y+x)$

$\Rightarrow 90x+9y=20y+2x$

$\Rightarrow 88x=11y$

$\Rightarrow 8x=y$

$\Rightarrow 8x-y=0$ ... **(2)**

Adding **(1)** and **(2)**, we get

$9x=9$

$\Rightarrow x=1$

Putting value of **x** in **(1)**, we get

$1+y=9$

$\Rightarrow y=9-1=8$

Therefore, number $=10x+y=10(1)+8=10+8=18$

(iv) Let number of Rs 100 notes $=x$and let number of Rs 50 notes $=y$

According to given conditions, we have

$x+y=25$ … **(1)**

and $100x+50y=2000$

$\Rightarrow 2x+y=40$ … **(2)**

Subtracting **(2)** from **(1)**, we get

$-x=-15$

$\Rightarrow x=15$

Putting value of **x** in **(1)**, we get

$15+y=25$

$\Rightarrow y=25-15=10$

Therefore, number of Rs 100 notes $=15$ and number of Rs 50 notes $=10$

(v) Let fixed charge for 3 days = Rs x

Let additional charge for each day thereafter = Rs y

According to given condition, we have

$x+4y=27$ … **(1)**

$x+2y=21$ … **(2)**

Subtracting **(2)** from **(1)**, we get

$2y=6$

$\Rightarrow y=3$

Putting value of **y** in **(1)**, we get

$x+4(3)=27$

$\Rightarrow x=27-12=15$

Therefore, fixed charge for 3 days = Rs 15 and additional charge for each day after 3 days = Rs 3

6. C of linear equations has unique solution, no solution, or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

(ii) $2x + y = 5$

$3x + 2y = 8$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Ans. (i) $x-3y-3=0$

$3x - 9y - 2 = 0$

Comparing equation $x - 3y - 3 = 0$ with $a_1x + b_1y + c_1 = 0$ and $3x - 9y - 2 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 1, b_1 = -3, c_1 = -3, a_2 = 3, b_2 = -9, c_2 = -2$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ this means that the two lines are parallel.

Therefore, there is no solution for the given equations i.e. it is inconsistent.

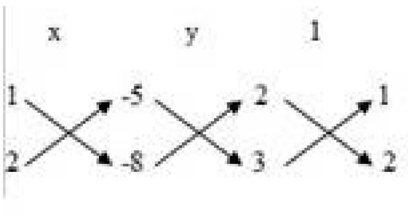
(ii) $2x + y = 5$

$3x + 2y = 8$

Comparing equation $2x + y = 5$ with $a_1x + b_1y + c_1 = 0$ and $3x + 2y = 8$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 2, b_1 = 1, c_1 = -5, a_2 = 3, b_2 = 2, c_2 = -8$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that there is unique solution for the given equations.



$$\frac{x}{(-8)(1)-(2)(-5)} = \frac{y}{(-5)(3)-(-8)(2)} = \frac{1}{(2)2-(3)1}$$

$$\Rightarrow \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$\Rightarrow x = 2$ and $y = 1$

(iii) $3x - 5y = 20$

$6x-10y=40$

Comparing equation $3x-5y=20$ with $a_1x+b_1y+c_1=0$ and $6x-10y=40$ with $a_2x+b_2y+c_2=0$,

We get $a_1= 3, b_1= -5, c_1=-20, a_2= 6, b_2= -10, c_2=-40$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

It means lines coincide with each other.

Hence, there are infinite many solutions.

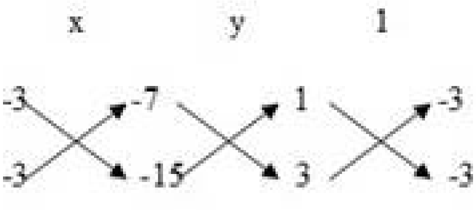
(iv) $x-3y-7=0$

$3x-3y-15=0$

Comparing equation $x-3y-7=0$ with $a_1x+b_1y+c_1=0$ and $3x-3y-15=0$ with $a_2x+b_2y+c_2=0$,

We get $a_1= 1, b_1= -3, c_1=-7, a_2= 3, b_2= -3, c_2=-15$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that we have unique solution for these equations.



$$\frac{x}{(-3)(-15)-(-3)(-7)} = \frac{y}{(-7)(3)-(-15)(1)} = \frac{1}{(-3)1-(-3)3}$$

$$\Rightarrow \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9}$$

$$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

⇒ $x = 4$ and $y = -1$

7. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Ans. (i) Let fixed monthly charge = Rs x and let charge of food for one day = Rs y

According to given conditions,

$x + 20y = 1000$ … (1),

and $x + 26y = 1180$ … (2)

Subtracting equation (1) from equation (2), we get

$6y = 180$

$\Rightarrow y = 30$

Putting value of y in (1), we get

$x + 20\,(30) = 1000$

$\Rightarrow x = 1000 - 600 = 400$

Therefore, fixed monthly charges = Rs 400 and, charges of food for one day = Rs 30

(ii) Let numerator = $x$and let denominator = y

According to given conditions,

$$\frac{x-1}{y} = \frac{1}{3} \quad \ldots(1) \quad \frac{x}{y+8} = \frac{1}{4} \quad \ldots(2)$$

$\Rightarrow 3x - 3 = y$ … (1) $4x = y + 8$ … (1)

$\Rightarrow 3x - y = 3$ … (1) $4x - y = 8$ … (2)

Subtracting equation (1) from (2), we get

$4x - y - (3x - y) = 8 - 3$

$\Rightarrow x = 5$

Putting value of x in (1), we get

$3\,(5) - y = 3$

$\Rightarrow 15 - y = 3$

$\Rightarrow y = 12$

Therefore, numerator = 5 and, denominator = 12

It means fraction = $\frac{x}{y} = \frac{5}{12}$

(iii) Let number of correct answers = x and let number of wrong answers = y

According to given conditions,

$3x - y = 40$ … (1)

And, $4x - 2y = 50$ … (2)

From equation (1), $y = 3x - 40$

Putting this in (2), we get

$4x - 2(3x - 40) = 50$

$\Rightarrow 4x - 6x + 80 = 50$

$\Rightarrow -2x = -30$

$\Rightarrow x = 15$

Putting value of x in (1), we get

$3(15) - y = 40$

$\Rightarrow 45 - y = 40$

$\Rightarrow y = 45 - 40 = 5$

Therefore, number of correct answers = x = 15and number of wrong answers = y = 5

Total questions = $x + y = 15 + 5 = 20$

(iv) Let speed of car which starts from part A = x km/hr

Let speed of car which starts from part B = y km/hr

According to given conditions,

$\frac{100}{x-y} = 5$ (Assuming x > y)

⇒ $5x - 5y = 100$

⇒ $x - y = 20$ … (1)

And, $\frac{100}{x+y} = 1$

⇒ $x + y = 100$ … (2)

Adding (1) and (2), we get

$2x = 120$

⇒ $x = 60$ km/hr

Putting value of x in (1), we get

$60 - y = 20$

⇒ $y = 60 - 20 = 40$ km/hr

Therefore, speed of car starting from point A = 60 km/hr

And, Speed of car starting from point B = 40 km/hr

(v) Let length of rectangle = x units and Let breadth of rectangle = y units

Area =xy square *units*. According to given conditions,

$xy - 9 = (x - 5)(y + 3)$

⇒ $xy - 9 = xy + 3x - 5y - 15$

⇒ $3x - 5y = 6$ … (1)

And, $xy + 67 = (x + 3)(y + 2)$

⇒ $xy + 67 = xy + 2x + 3y + 6$

$\Rightarrow 2x + 3y = 61$ … (2)

From equation (1),

$3x = 6 + 5y$

$\Rightarrow x = \frac{6+5y}{3}$

Putting this in (2), we get

$2\left(\frac{6+5y}{3}\right) + 3y = 61$

$\Rightarrow 12 + 10y + 9y = 183$

$\Rightarrow 19y = 171$

$\Rightarrow y = 9$ units

Putting value of y in (2), we get

$2x + 3(9) = 61$

$\Rightarrow 2x = 61 - 27 = 34$

$\Rightarrow x = 17$ units

Therefore, length = 17 units and, breadth = 9 units

8. Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x} + \frac{1}{3y} = 2$

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14$

$\frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=1$

(v) $7x-2y=5xy$

$8x+7y=15xy$

(vi) $6x+3y-6xy=0$

$2x+4y-5xy=0$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$

Ans. (i) $\frac{1}{2x}+\frac{1}{3y}=2$ ... (1)

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$ ... (2)

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

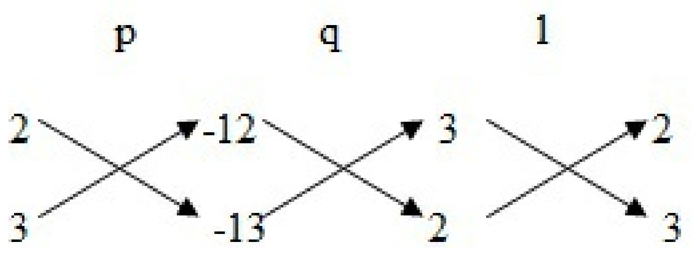
Putting this in equation (1) and (2), we get

$\frac{p}{2}+\frac{q}{3}=2 \; and \; \frac{p}{3}+\frac{q}{2}=\frac{13}{6}$

⇒ $3p+2q=12$ and $6(2p+3q)=13(6)$

⇒ $3p+2q=12$ and $2p + 3q =13$

⇒ $3p+2q-12=0$ ... (3) and $2p+3q-13=0$ ... (4)



$$\frac{p}{2(-13)-3(-12)}=\frac{q}{(-12)2-(-13)3}=\frac{1}{3\times3-2\times2}$$

$$\Rightarrow \frac{p}{-26+36}=\frac{q}{-24+39}=\frac{1}{9-4}$$

$$\Rightarrow \frac{p}{10}=\frac{q}{15}=\frac{1}{5}$$

$$\Rightarrow \frac{p}{10}=\frac{1}{5} \; and \; \frac{q}{15}=\frac{1}{5}$$

$\Rightarrow p=2$ and $q=3$

But $\frac{1}{x}=p$ and $\frac{1}{y}=q$

Putting value of p and q in this we get

$x=\frac{1}{2}$ and $y=\frac{1}{3}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$ ... (1)

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$ ... (2)

Let $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$

Putting this in (1) and (2), we get

$2p+3q=2$ ... (3)

$4p-9q=-1$ ... (4)

Multiplying (3) by 2 and subtracting it from (4), we get

$4p-9q+1-2(2p+3q-2)=0$

$\Rightarrow 4p-9q+1-4p-6q+4=0$

$\Rightarrow -15q+5=0$

$\Rightarrow q=\frac{-5}{-15}=\frac{1}{3}$

Putting value of q in (3), we get

$2p+1=2$

$\Rightarrow 2p=1 \Rightarrow p=\frac{1}{2}$

Putting values of p and q in ($\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$), we get

$$\frac{1}{\sqrt{x}}=\frac{1}{2} \text{ and } \frac{1}{\sqrt{y}}=\frac{1}{3}$$

$$\Rightarrow \frac{1}{x}=\frac{1}{4} \text{ and } \frac{1}{y}=\frac{1}{9}$$

$\Rightarrow x=4$ and $y=9$

(iii) $\frac{4}{x}+3y=14$ … (1)

$\frac{3}{x}-4y=23$ … (2) and Let $\frac{1}{x}=p$ … (3)

Putting (3) in (1) and (2), we get

$4p+3y=14$ … (4)

$3p-4y=23$ … (5)

Multiplying (4) by 3 and (5) by 4, we get

$3(4p+3y-14=0)$ and, $4(3p-4y-23=0)$

$\Rightarrow 12p+9y-42=0$ … (6) $12p-16y-92=0$ … (7)

Subtracting (7) from (6), we get

$9y-(-16y)-42-(-92)=0$

$\Rightarrow 25y+50=0$

$\Rightarrow y=50-25=-2$

Putting value of y in (4), we get

$4p+3(-2)=14$

$\Rightarrow 4p-6=14$

$\Rightarrow 4p=20$

$\Rightarrow p=5$

Putting value of p in (3), we get

$\frac{1}{x}=5$

$\Rightarrow x=\frac{1}{5}$

Therefore, $x=\frac{1}{5}$ and $y=-2$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$ ... (1)

$\frac{6}{x-1}-\frac{3}{y-2}=1$ ... (2)

Let $\frac{1}{x-1}=p$ *and* $\frac{1}{y-2}=q$

Putting this in (1) and (2), we get

$5p+q=2$

$\Rightarrow 5p+q-2=0$ ... (3)

And, $6p-3q=1$

$\Rightarrow 6p-3q-1=0$ ... (4)

Multiplying (3) by 3 and adding it to (4), we get

$3(5p+q-2)+6p-3q-1=0$

$\Rightarrow 15p+3q-6+6p-3q-1=0$

$\Rightarrow 21p-7=0$

$\Rightarrow p=\frac{1}{3}$

Putting this in (3), we get

$5(\frac{1}{3})+q-2=0$

$\Rightarrow 5+3q=6$

$\Rightarrow 3q=6-5=1$

$\Rightarrow q=\frac{1}{3}$

Putting values of p and q in ($\frac{1}{x-1}=p \ \ and \ \frac{1}{y-2}=q$), we get

$\frac{1}{x-1}=\frac{1}{3} and \frac{1}{y-2}=\frac{1}{3}$

$\Rightarrow 3=x-1$ and $3=y-2$

$\Rightarrow x=4$ and $y=5$

(v) $7x-2y=5xy$ … (1)

$8x+7y=15xy$ … (2)

Dividing both the equations by xy, we get

$\frac{7}{y}-\frac{2}{x}=5$ …(3)

$\frac{8}{y}+\frac{7}{x}=15$ ...(4)

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

Putting these in (3) and (4), we get

$7q-2p=5$ ... (5)

$8q+7p=15$ ... (6)

From equation (5),

$2p=7q-5$

$\Rightarrow p=\frac{7q-5}{2}$

Putting value of p in (6), we get

$8q+7\left(\frac{7q-5}{2}\right)=15$

$\Rightarrow 16q+49q-35=30$

$\Rightarrow 65q=30+35=65$

$\Rightarrow q=1$

Putting value of q in (5), we get

$7(1)-2p=5$

$\Rightarrow 2p=2 \Rightarrow p=1$

Putting value of p and q in ($\frac{1}{x}=p$ and $\frac{1}{y}=q$), we get $x=1$ and $y=1$

(vi) $6x+3y-6xy=0$ ... (1)

$2x+4y-5xy=0$ ... (2)

Dividing both the equations by xy, we get

$$\frac{6}{y}+\frac{3}{x}-6=0 \quad ...(3)$$

$$\frac{2}{y}+\frac{4}{x}-5=0 \quad ...(4)$$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

Putting these in (3) and (4), we get

$6q+3p-6=0$ ... (5)

$2q+4p-5=0$ ... (6)

From (5),

$3p=6-6q$

$\Rightarrow p=2-2q$

Putting this in (6), we get

$2q+4(2-2q)-5=0$

$\Rightarrow 2q+8-8q-5=0$

$\Rightarrow -6q=-3$

$\Rightarrow q=\frac{1}{2}$

Putting value of q in (p=2 – 2q), we get

$p=2-2\left(\frac{1}{2}\right)=2-1=1$

Putting values of p and q in ($\frac{1}{x}=p$ and $\frac{1}{y}=q$), we get $x=1$ and $y=2$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$ ... (1)

$\frac{15}{x+y}-\frac{5}{x-y}=-2$ ... (2)

Let $\frac{1}{x+y}=p \text{ and } \frac{1}{x-y}=q$

Putting this in (1) and (2), we get

$10p+2q=4$ ... (3)

$15p-5q=-2$ ... (4)

From equation (3),

$2q=4-10p$

$\Rightarrow q=2-5p$ ... (5)

Putting this in (4), we get

$15p-5(2-5p)=-2$

$\Rightarrow 15p-10+25p=-2$

$\Rightarrow 40p=8$

$\Rightarrow p=\frac{1}{5}$

Putting value of p in (5), we get

$q=2-5(\frac{1}{5})=2-1=1$

Putting values of p and q in ($\frac{1}{x+y}=p \text{ and } \frac{1}{x-y}=q$),we get

$$\frac{1}{x+y}=\frac{1}{5} \text{ and } \frac{1}{x-y}=\frac{1}{1}$$

⇒ $x+y=5$ … (6) and $x-y=1$ … (7)

Adding (6) and (7), we get

$2x=6$

⇒ $x=3$

Putting $x=3$ in (7), we get

$3-y=1$

⇒ $y=3-1=2$

Therefore, $x=3$ and $y=2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$ … (1)

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$ … (2)

Let $\frac{1}{3x+y}=p \text{ and } \frac{1}{3x-y}=q$

Putting this in (1) and (2), we get

$p+q=\frac{3}{4}$ and $\frac{p}{2}-\frac{q}{2}=-\frac{1}{8}$

⇒ $4p+4q=3$ … (3) and $4p-4q=-1$ … (4)

Adding (3) and (4), we get

$8p=2$

⇒ $p=\frac{1}{4}$

Putting value of p in (3), we get

$4\,(\frac{1}{4})+4q=3$

$\Rightarrow 1+4q=3$

$\Rightarrow 4q=3-1=2$

$\Rightarrow q=\frac{1}{2}$

Putting value of p and q in $\frac{1}{3x+y}=p$ and $\frac{1}{3x-y}=q$, we get

$$\frac{1}{3x+y}=\frac{1}{4} \text{ and } \frac{1}{3x-y}=\frac{1}{2}$$

$\Rightarrow 3x+y=4$ … (5) and $3x-y=2$ … (6)

Adding (5) and (6), we get

$6x=6$

$\Rightarrow x=1$

Putting $x=1$ in (5) , we get

$3(1)+y=4$

$\Rightarrow y=4-3=1$

Therefore, $x=1$ and $y=1$

9. Formulate the following problems as a part of equations, and hence find their solutions.

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish

the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Ans. (i) Let speed of rowing in still water $=x$ km/h

Let speed of current $=y$ km/h

So, speed of rowing downstream $=(x+y)$ km/h

And, speed of rowing upstream $=(x-y)$ km/h

According to given conditions,

$$\frac{20}{x+y}=2 \text{ and } \frac{4}{x-y}=2$$

$\Rightarrow 2x+2y=20$ and $2x-2y=4$

$\Rightarrow x+y=10$ ... (1) and $x-y=2$ ... (2)

Adding (1) and (2), we get

$2x=12$

$\Rightarrow x=6$

Putting $x=6$ in (1), we get

$6+y=10$

$\Rightarrow y=10-6=4$

Therefore, speed of rowing in still water $=6$ km/h

Speed of current $=4$ km/h

(ii) Let time taken by 1 woman alone to finish the work $=x$ days

Let time taken by 1 man alone to finish the work =*y* days

So, 1 woman's 1 day work =($\frac{1}{x}$)*th* part of the work

And, 1 man's 1 day work =($\frac{1}{y}$)*th* part of the work

So, 2 women's 1 day work =($\frac{2}{x}$)*th* part of the work

And, 5 men's 1 day work =($\frac{5}{y}$)*th* part of the work

Therefore, 2 women and 5 men's 1 day work =($\frac{2}{x}+\frac{5}{y}$)*th* part of the work... (1)

It is given that 2 women and 5 men complete work in = 4 days

It means that in 1 day, they will be completing $\frac{1}{4}$ *th* part of the work ... (2)

Clearly, we can see that (1) = (2)

$\Rightarrow \frac{2}{x}+\frac{5}{y}=\frac{1}{4}$ ... (3)

Similarly, $\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$ ... (4)

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

Putting this in (3) and (4), we get

$2p+5q=\frac{1}{4}$ and $3p+6q=\frac{1}{3}$

$\Rightarrow 8p+20q=1$ ... (5) and $9p+18q=1$ ... (6)

Multiplying (5) by 9 and (6) by 8, we get

$72p+180q=9$ ... (7)

$72p+144q=8$ ... (8)

Subtracting (8) from (7), we get

$36q=1$

$\Rightarrow q=\frac{1}{36}$

Putting this in (6), we get

$9p+18(\frac{1}{36})=1$

$\Rightarrow 9p=\frac{1}{2}$

$\Rightarrow p=\frac{1}{18}$

Putting values of p and q in $\frac{1}{x}=p \text{ and } \frac{1}{y}=q$, we get $x=18$ and $y=36$

Therefore, 1 woman completes work in =18 days

And, 1 man completes work in =36 days

(iii) Let speed of train $=x$ km/h and let speed of bus $=y$ km/h

According to given conditions,

$$\frac{60}{x}+\frac{240}{y}=4 \text{ and } \frac{100}{x}+\frac{200}{y}=4+\frac{10}{60}$$

Let $\frac{1}{x}=p \text{ and } \frac{1}{y}=q$

Putting this in the above equations, we get

$60p+240q=4$ ... (1)

And $100p+200q=\frac{25}{6}$ ... (2)

Multiplying (1) by 5 and (2) by 3, we get

$300p+1200q=20$ ... (3)

$300p+600q=\frac{25}{2}$ ... (4)

Subtracting (4) from (3), we get

$600q=20-\frac{25}{2}=7.5$

$\Rightarrow q=\frac{7.5}{600}$

Putting value of q in (2), we get

$100p+200(\frac{7.5}{600})=\frac{25}{6}$

$\Rightarrow 100p+2.5=\frac{25}{6}$

$\Rightarrow 100p=\frac{25}{6}-2.5$

$\Rightarrow p=\frac{10}{600}$

But $\frac{1}{x}=p\ and\ \frac{1}{y}=q$

Therefore, $x = \frac{600}{10} = 60$ km/h and $y = \frac{600}{7.5} = 80$ km/h

Therefore, speed of train =60 km/h

And, speed of bus =80 km/h

10. Solve the following pair of linear equations:

(i) $px + py = p - q$

$qx - py = p + q$

(ii) $ax + by = c$

$bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

(iv) $(a-b)x + (a+b)y = a^2 - 2ab - b^2$

$(a+b)(x+y) = a^2 + b^2$

(v) $152x - 378y = -74$

$-378x + 152y = -604$

Ans. (i) $px + qy = p - q$ ... (1)

$qx - py = p + q$ ... (2)

Multiplying equation (1) by p and equation (2) by q, we obtain:

$p^2x + pqy = p^2 - pq$ ... (3)

$q^2x - pqy = pq + q^2$ ... (4)

Adding equations (3) and (4), we obtain:

$p^2x + q^2x = p^2 + q^2$

$$\Rightarrow \left(p^2+q^2\right)x = p^2+q^2$$

$$\Rightarrow x = \frac{p^2+q^2}{p^2+q^2} = 1$$

Substituting the value of x in equation (1), we obtain:

$$p(1)+qy = p-q$$

$$\Rightarrow qy = -q$$

$$\Rightarrow y = -1$$

Hence the required solution is x = 1 and y = –1.

(ii) $ax+by=c$ ... (1)

$bx+ay=1+c$ ... (2)

Multiplying equation (1) by a and equation (2) by b, we obtain:

$a^2x+aby=ac$ ... (3)

$b^2x+aby=b+bc$ ... (4)

Subtracting equation (4) from equation (3),

$$\left(a^2-b^2\right)x = ac-bc-b$$

$$\Rightarrow x = \frac{c(a-b)-b}{a^2-b^2}$$

Substituting the value of x in equation (1), we obtain:

$$a\left\{\frac{c(a-b)-b}{a^2-b^2}\right\}+by = c$$

$$\Rightarrow \frac{ac(a-b)-ab}{a^2-b^2}+by = c$$

$$\Rightarrow by = c-\frac{ac(a-b)-ab}{a^2-b^2}$$

$$\Rightarrow by = \frac{a^2c - b^2c - a^2c + abc + ab}{a^2 - b^2}$$

$$\Rightarrow by = \frac{abc - b^2c + ab}{a^2 - b^2}$$

$$\Rightarrow y = \frac{c(a-b)+a}{a^2 - b^2}$$

(iii) $\frac{x}{a} - \frac{y}{b} = 0 \Rightarrow bx - ay = 0$ ....(1)

$ax + by = a^2 + b^2$ ....(2)

Multiplying equation (1) and (2) by b and a respectively, we obtain:

$b^2x - aby = 0$ ....(3)

$a^2x + aby = a^3 + ab^2$ ....(4)

Adding equations (3) and (4), we obtain:

$b^2x + a^2x = a^3 + ab^2$

$\Rightarrow x(b^2 + a^2) = a(a^2 + b^2)$

$\Rightarrow x = a$

Substituting the value of x in equation (1), we obtain:

$b(a) - ay = 0$

$\Rightarrow ab - ay = 0$

$\Rightarrow y = b$

(iv) $(a-b)x + (a+b)y = a^2 - 2ab - b^2$ ... (1)

$(a+b)(x+y) = a^2 + b^2$

$\Rightarrow (a+b)x+(a+b)y=a^2+b^2$ ....(2)

Subtracting equation (2) from (1), we obtain:

$(a-b)x-(a+b)x=(a^2-2ab-b^2)-(a^2+b^2)$

$\Rightarrow (a-b-a-b)x=-2ab-2b^2$

$\Rightarrow -2bx=-2b(a+b)$

$\Rightarrow x=a+b$

Substituting the value of x in equation (1), we obtain:

$(a-b)(a+b)+(a+b)y=a^2-2ab-b^2$

$\Rightarrow a^2-b^2+(a+b)y=a^2-2ab-b^2$

$\Rightarrow (a+b)y=-2ab$

$\Rightarrow y=\dfrac{-2ab}{a+b}$

(v) 152x – 378y = –74 ... (1)

–378x + 152y = –604 ... (2)

Adding the equations (1) and (2), we obtain:

–226x – 226y = –678

$\Rightarrow$ x + y = 3 ....(3)

Subtracting the equation (2) from equation (1), we obtain:

530x – 530y = 530

$\Rightarrow$ x – y = 1 ....(4)

Adding equations (3) and (4), we obtain:

2x = 4

$\Rightarrow$ x = 2

Substituting the value of x in equation (3), we obtain:

y = 1

11. Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi he takes 6 minute longer. Find the speed of the train and that of the taxi

Ans. Let the speed of the train be x and taxi be y km/h

According to question,

$$\frac{300}{x}+\frac{200}{y}=5\frac{1}{2}$$

$$\text{or } \frac{300}{x}+\frac{200}{y}=\frac{11}{2} \rightarrow (i)$$

$$\text{and } \frac{260}{x}+\frac{240}{y}=\left(\frac{11}{2}+\frac{1}{10}\right)$$

$$\text{or } \frac{260}{x}+\frac{240}{y}=\frac{56}{10} \rightarrow (ii)$$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$

$$\therefore\ 300u+200v=\frac{11}{2} \rightarrow (iii)$$

$$\text{and } 260u+240v=\frac{56}{10} \rightarrow (iv)$$

on solving eq (iii) and (iv), we get,

$$u=\frac{1}{100} \Rightarrow x=1000km/h$$

$$v=\frac{1}{80} \Rightarrow y=80km/h$$

12. If in a rectangle the length is increased and breadth is decreased by 2 units each, the area is reduced by 28 square units, and if the length is reduced by 1 unit and breadth is increased by 2 units, the area increased by 33 square units. Find the dimensions of the

rectangle.

Ans. Let the length and breadth of a rectangle be x and y meters.

According to question,

$Area = xy$

$(x+2)(y-2) = xy - 28$

$or\ 2x - 2y = 24$

$or\ x - y = 12 \rightarrow (i)$

and $(x-1)(y+2) = xy + 33$

$2x - y = 33 \rightarrow (ii)$

on subtracting eq (ii) − (i), we get,

$x = 21$ and

$21 - y = 12$

$so\ y = 21 - 12$

$y = 9$

$\therefore$ length $= 21m$

breadth $= 9m$

13. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$

$2x - 3y = 7$

(ii) $2x - 3y = 8$

$4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

$9x - 10y = 14$

(iv) $5x - 3y = 11$

$-10x + 6y = -2$

Ans. (i) $3x+2y=5$, $2x-3y=7$

Comparing equation $3x+2y=5$ with $a_1x+b_1y+c_1=0$and $2x-3y=7$ with $a_2x+b_2y+c_2=0$,

We get, $a_1= 3, b_1= 2, c_1= 5$, $a_2= 2, b_2= -3, c_2= -7$

$$\frac{a_1}{a_2}=\frac{3}{2} \text{ and } \frac{b_1}{b_2}=\frac{2}{3}$$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ which means equations have unique solution.

Hence they are consistent.

(ii) $2x-3y=8$, $4x-6y=9$

Comparing equation $2x-3y=8$ with $a_1x+b_1y+c_1=0$and $4x-6y=9$ with $a_2x+b_2y+c_2=0$,

We get, $a_1= 2, b_1= -3, c_1= -8$, $a_2= 4, b_2= -6, c_2= -9$

Here $\frac{a_1}{a_2}=\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ because $\frac{1}{2}=\frac{1}{2} \neq \frac{-8}{-9}$

Therefore, equations have no solution because they are parallel.

Hence, they are inconsistent.

(iii) $\frac{3}{2}x+\frac{5}{3}y=7$, $9x - 10y = 14$

Comparing equation $\frac{3}{2}x+\frac{5}{3}y=7$ with $a_1x+b_1y+c_1=0$and $9x - 10y = 14$ with $a_2x+b_2y+c_2=0$, We get, $a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = -14, a_2 = 9, b_2 = -10, c_2 = -14$

$$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6} \text{ and } \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{-1}{6}$$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, equations have unique solution.

Hence, they are consistent.

(iv) $5x-3y=11, -10x+6y=-22$

Comparing equation $5x-3y=11$ with $a_1x+b_1y+c_1=0$and $-10x+6y=-22$ with $a_2x+b_2y+c_2=0$,

We get, $a_1 = 5, b_1 = -3, c_1 = -11, a_2 = -10, b_2 = 6, c_2 = 22$

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2} \text{ and } \frac{c_1}{c_2} = \frac{11}{-22} = \frac{-1}{2}$$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the lines have infinite many solutions.

Hence, they are consistent.

14. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) x – 3y – 3 = 0

3x – 9y – 2 = 0

(ii) $2x + y = 5$

$3x + 2y = 8$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Ans. (i) $x-3y-3=0$

$3x-9y-2=0$

Comparing equation $x-3y-3=0$ with $a_1x+b_1y+c_1=0$ and $3x-9y-2=0$ with $a_2x+b_2y+c_2=0$,

We get $a_1=1, b_1=-3, c_1=-3, a_2=3, b_2=-9, c_2=-2$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ this means that the two lines are parallel.

Therefore, there is no solution for the given equations i.e. it is inconsistent.

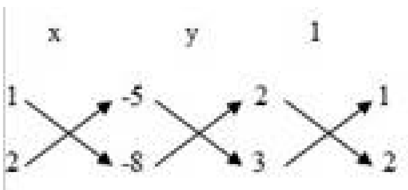
(ii) $2x+y=5$

$3x+2y=8$

Comparing equation $2x+y=5$ with $a_1x+b_1y+c_1=0$ and $3x+2y=8$ with $a_2x+b_2y+c_2=0$,

We get $a_1=2, b_1=1, c_1=-5, a_2=3, b_2=2, c_2=-8$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that there is unique solution for the given equations.



$$\frac{x}{(-8)(1)-(2)(-5)}=\frac{y}{(-5)(3)-(-8)(2)}=\frac{1}{(2)2-(3)1}$$

$$\Rightarrow \frac{x}{-8+10}=\frac{y}{-15+16}=\frac{1}{4-3}$$

$$\Rightarrow \frac{x}{2}=\frac{y}{1}=\frac{1}{1}$$

$\Rightarrow x=2$ and $y=1$

(iii) $3x-5y=20$

$6x-10y=40$

Comparing equation $3x-5y=20$ with $a_1x+b_1y+c_1=0$ and $6x-10y=40$ with $a_2x+b_2y+c_2=0$,

We get$a_1=3, b_1=-5, c_1=-20, a_2=6, b_2=-10, c_2=-40$

Here $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

It means lines coincide with each other.

Hence, there are infinite many solutions.

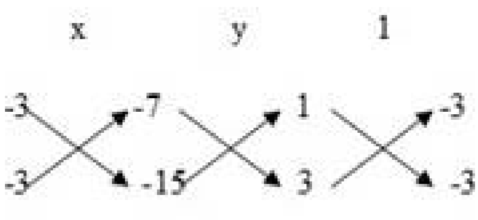
(iv) $x-3y-7=0$

$3x-3y-15=0$

Comparing equation $x-3y-7=0$ with $a_1x+b_1y+c_1=0$ and $3x-3y-15=0$ with $a_2x+b_2y+c_2=0$,

We get $a_1=1, b_1=-3, c_1=-7, a_2=3, b_2=-3, c_2=-15$

Here $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ this means that we have unique solution for these equations.



$$\frac{x}{(-3)(-15)-(-3)(-7)} = \frac{y}{(-7)(3)-(-15)(1)} = \frac{1}{(-3)1-(-3)3}$$

$$\Rightarrow \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9}$$

$$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$\Rightarrow x = 4$ and $y = -1$

15. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$8x+5y=9$

$3x+2y=4$

Ans. Substitution Method

$8x+5y=9$ ... **(1)**

$3x+2y=4$ ... **(2)**

From equation **(1)**,

$5y=9-8x$

$\Rightarrow y = \frac{9-8x}{5}$

Putting this in equation **(2)**, we get

$$3x+2\left(\frac{9-8x}{5}\right)=4$$

$$\Rightarrow 3x+\frac{18-16x}{5}=4$$

$$\Rightarrow 3x-\frac{16}{5}x=\frac{4}{1}-\frac{18}{5}$$

$\Rightarrow 15x-16x=20-18$

$\Rightarrow x=-2$

Putting value of **x** in **(1)**, we get

$8(-2)+5y=9$

$\Rightarrow 5y=9+16=25$

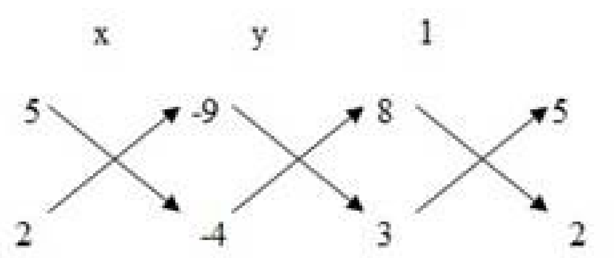
$\Rightarrow y=5$

Therefore, $x=-2$ and $y=5$

Cross multiplication method

$8x+5y=9$ ... **(1)**

$3x+2y=4$ ... **(2)**



$$\frac{x}{5(-4)-2(-9)}=\frac{y}{(-9)3-(-4)8}=\frac{1}{8\times 2-5\times 3}$$

$$\Rightarrow \frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$$

$$\Rightarrow \frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$\Rightarrow x=-2$ and $y=5$

16. In a $\triangle$ABC, $\angle$C = 3$\angle$B = 2($\angle$A + $\angle$B). Find three angles

Ans. $\angle C = 3\angle B = 2(\angle A + \angle B)$

Taking $3\angle B = 2(\angle A + \angle B)$

$\Rightarrow \angle B = 2\angle A$

$\Rightarrow 2\angle A - \angle B = 0$ ....(1)

We know that the sum of the measures of all angles of a triangle is 180°.

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle B + 3\angle B = 180°$

$\Rightarrow \angle A + 4\angle B = 180°$ ....(2)

Multiplying equation (1) by 4, we obtain:

$8\angle A - 4\angle B = 0$ ....(3)

Adding equations (2) and (3), we get

$9\angle A = 180°$

$\Rightarrow \angle A = 20°$

From eq. (2), we get,

$20° + 4\angle B = 180°$

$\Rightarrow \angle B = 40°$

And $\angle C = 3 \times 40° = 120°$

Hence the measures of $\angle$A, $\angle$B and $\angle$C are 20°, 40° and 120° respectively.